1. Given that Gaussian is given by \( g(x, y) = e^{-\frac{(x^2 + y^2)}{2\sigma^2}} \) derive Laplacian of Gaussian:

\[
\Delta G_{\sigma} = -\frac{1}{\sqrt{2\pi\sigma^2}} \left( 2 - \frac{x^2 + y^2}{\sigma^2} \right) e^{-\frac{x'^2 + y'^2}{2\sigma^2}}
\]

2. Show that Laplacian \( \Delta^2 I = I_{xx} + I_{yy} \) can also be written as \( \Delta^2 = I_{\theta\theta} + I_{nn} \), where \( I_{\theta\theta} \) is second derivative in direction \( \theta \) and \( I_{nn} \) is second derivative in direction perpendicular to \( \theta \).

3. Show that Harris corner detector is rotation invariant.

4. Show that gradient magnitude is rotation invariant: \( \sqrt{I_x^2 + I_y^2} = \sqrt{I_{x'}^2 + I_{y'}^2} \), where \( (x', y') \) are rotated coordinates of \( (x, y) \).

5. If Taylor series expansion of DOG (Difference of Gaussian) is given by

\[
D(x) = D + \frac{\partial D^T}{\partial x} x + \frac{1}{2} x^T \frac{\partial^2 D}{\partial x^2} x
\]

Show that the location and scale of minima/maxima is given by

\[
\dot{x} = -\frac{\partial^2 D^{-1}}{\partial x^2} \frac{\partial D}{\partial x}.
\]