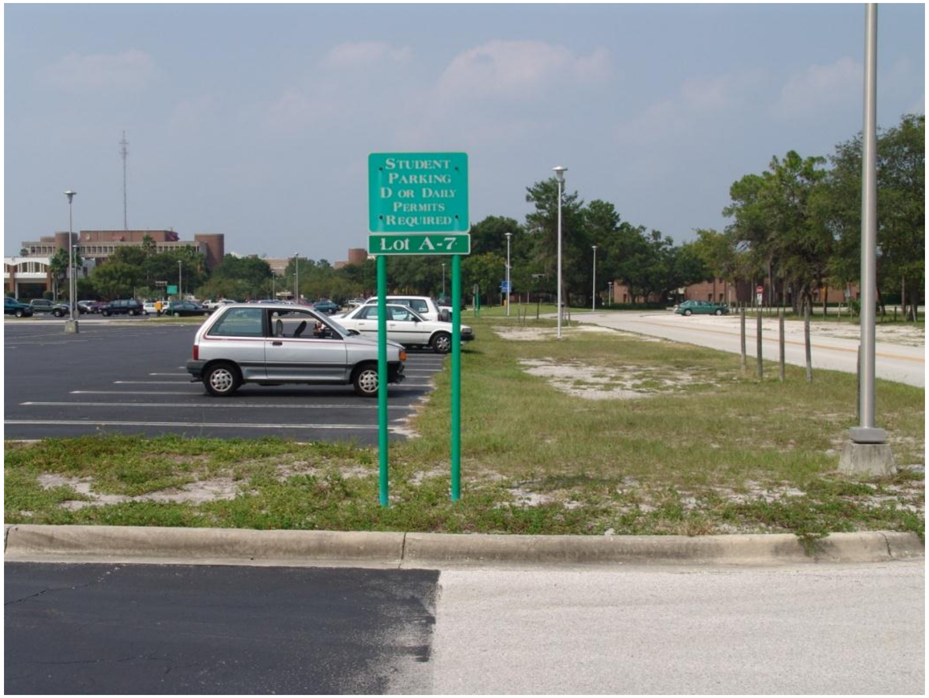
Camera Model and Calibration

Lecture-12

Camera Calibration

- Determine extrinsic and intrinsic parameters of camera
  - Extrinsic
    - 3D location and orientation of camera
  - Intrinsic
    - Focal length
    - The size of the pixels

Application: Object Transfer



Source Image

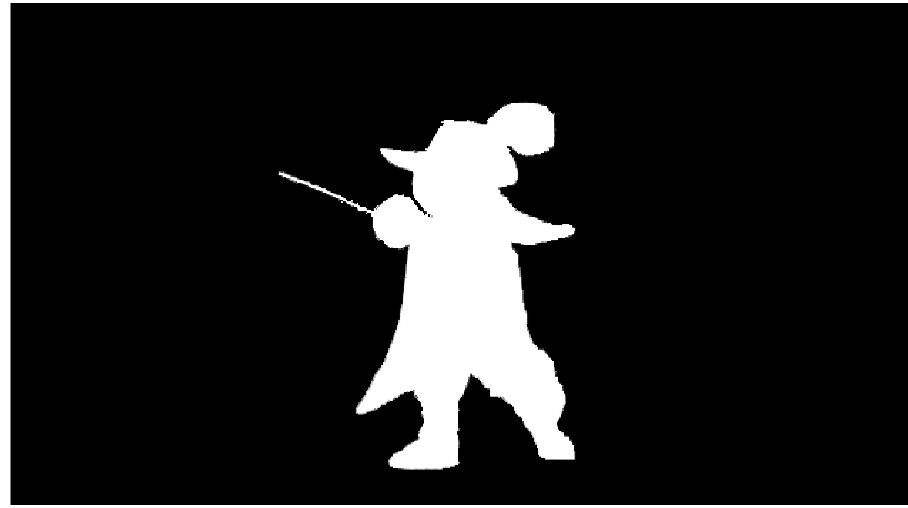
Target Image

More Results

Source Image

Target Image

More Results



Application in Film Industry

Pose Estimation

- Given 3D model of object, and its image (2D projection) determine the location and orientation (translation & rotation) of object such that when projected on the image plane it will match with the image.

http://www.youtube.com/watch?v=ZNHRH00UMvk

Transformations

3-D Translation

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = T \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Translation Matrix}$$

$$\mathrm{T}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -d_x \\ 0 & 1 & 0 & -d_y \\ 0 & 0 & 1 & -d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$TT^{-1} = T^{-1}T = I$$

$$\begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -d_x \\ 0 & 1 & 0 & -d_y \\ 0 & 0 & 1 & -d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_1 \times S_x \\ Y_1 \times S_y \\ Z_1 \times S_z \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1/S_x & 0 & 0 & 0 \\ 0 & 1/S_y & 0 & 0 \\ 0 & 0 & 1/S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$SS^{-1} = S^{-1}S = I$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = S \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

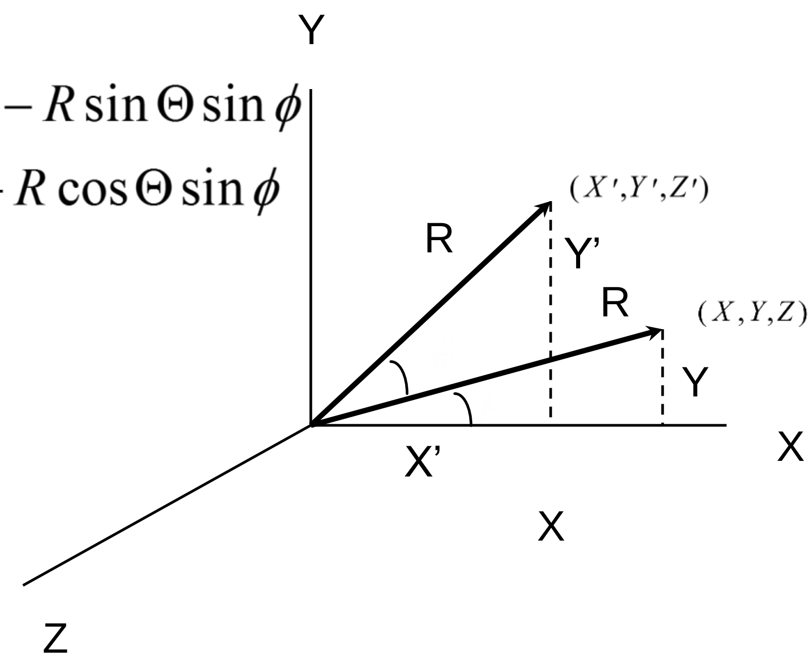
$$S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Scaling Matrix}$$

$$\left[\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/S_x & 0 & 0 & 0 \\ 0 & 1/S_y & 0 & 0 \\ 0 & 0 & 1/S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation

$$X = R\cos\phi$$
$$Y = R\sin\phi$$

$$X' = R\cos(\Theta+\phi) = R\cos\Theta\cos\phi - R\sin\Theta\sin\phi$$
$$Y' = R\sin(\Theta+\phi) = R\sin\Theta\cos\phi + R\cos\Theta\sin\phi$$



$$X' = X\cos\Theta - Y\sin\Theta$$
$$Y' = X\sin\Theta + Y\cos\Theta$$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$(R_\theta^Z)^{-1} = \begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(R_\theta^Z)^{-1} = (R_\theta^Z)^T$$

Rotation matrices are orthonormal matrices

$$(R_\theta^Z)(R_\theta^Z)^T = I$$

$$r_i . r_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

Euler Angles

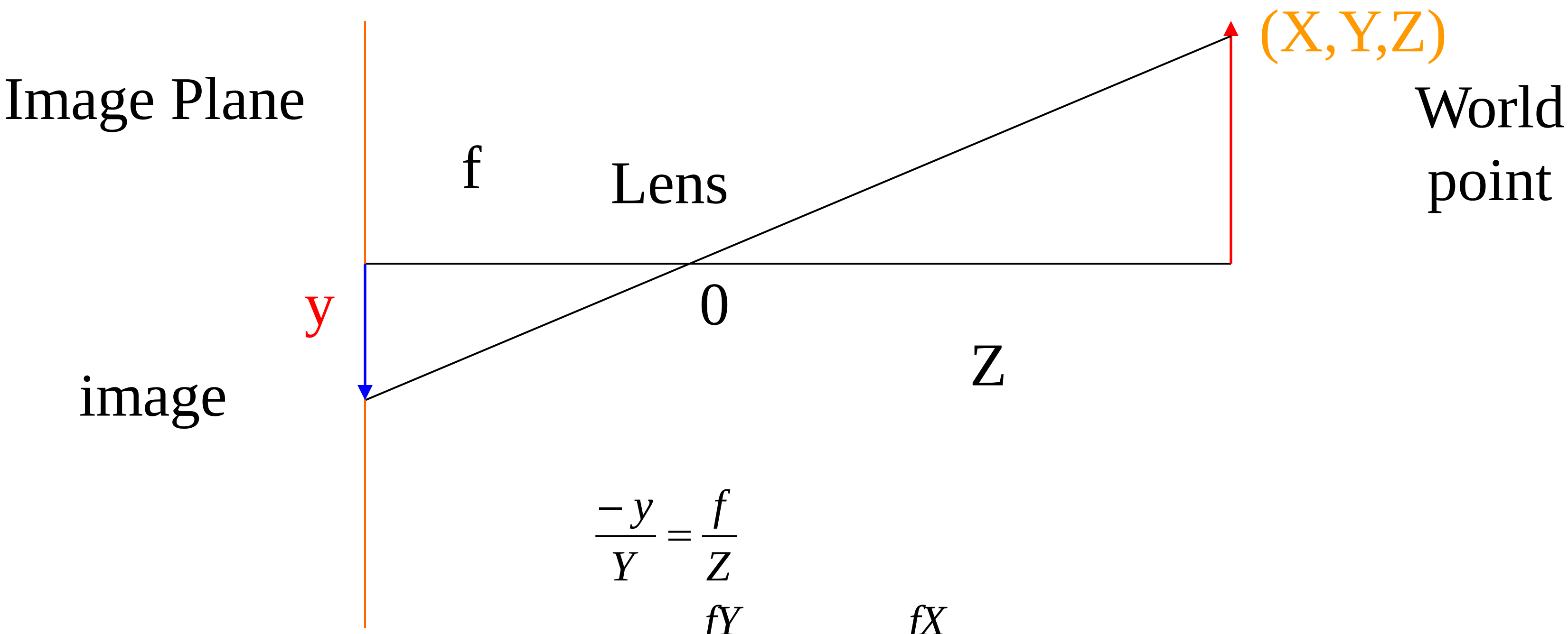
Rotation around an arbitrary axis:

$$R = R_Z^{\alpha} R_Y^{\beta} R_X^{\gamma} = \begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix}$$

↓ if angles are small $\cos\Theta \approx 1$ $\sin\Theta \approx \Theta$

$$R = \begin{bmatrix} 1 & -\alpha & \beta \\ \alpha & 1 & -\gamma \\ -\beta & \gamma & 1 \end{bmatrix}$$

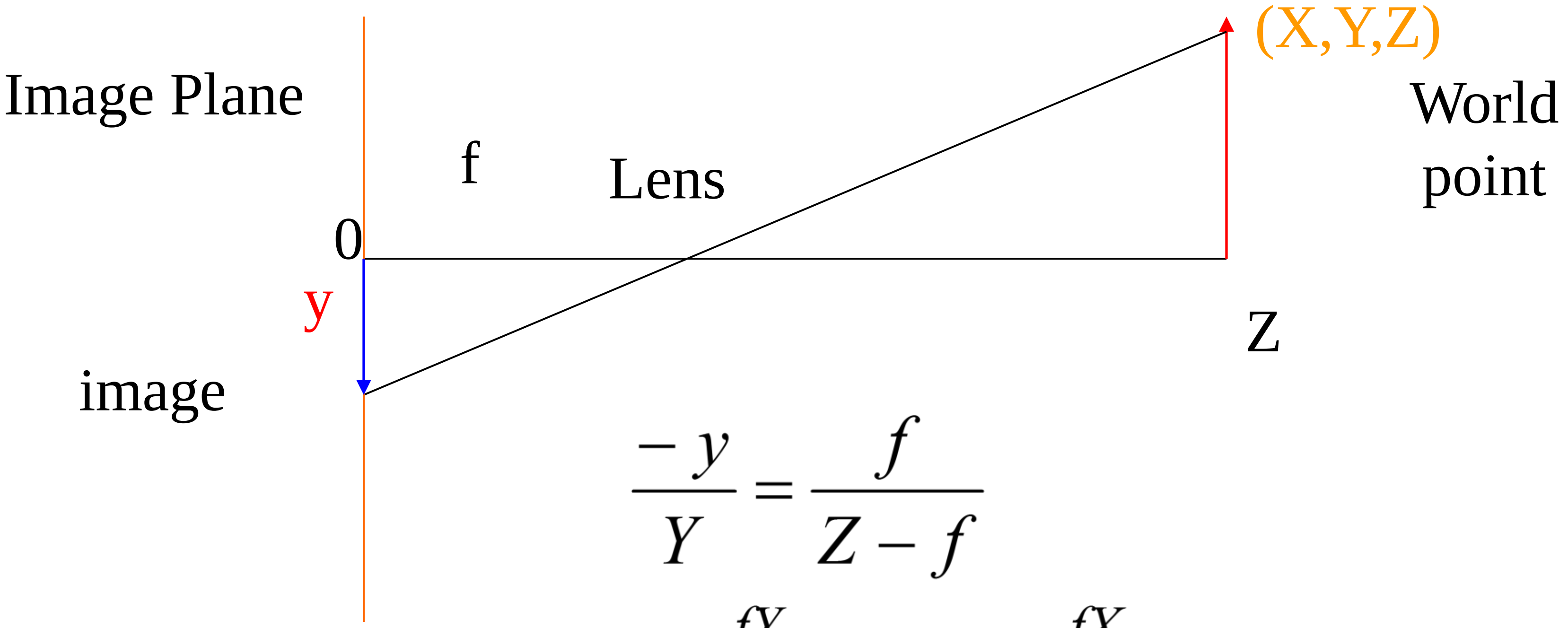
Perspective Projection (origin at the lens center)



$$\frac{-y}{Y} = \frac{f}{Z}$$

$$y = -\frac{fY}{Z} \qquad x = -\frac{fX}{Z}$$

Perspective Projection (origin at image center)



$$\frac{-y}{Y} = \frac{f}{Z-f}$$

$$y = -\frac{fY}{Z-f} \qquad x = -\frac{fX}{Z-f}$$

$$y = \frac{fY}{f-Z} \qquad x = \frac{fX}{f-Z}$$

Perspective

Image coordinates

$$\begin{bmatrix} x \\ y \\ \\ \end{bmatrix} = \begin{bmatrix} & ? & \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

World coordinates

$$(X, Y, Z) \rightarrow \rightarrow\rightarrow \qquad (kX, kY, kZ, k), \text{Homogenous transformation}$$

$$(\mathrm{C}_{\mathrm{h1}}, C_{h2}, C_{h3}, C_{h4}) \rightarrow \qquad (\frac{\mathrm{C}_{\mathrm{h1}}}{C_{h4}}, \frac{C_{h2}}{C_{h4}}, \frac{C_{h3}}{C_{h4}}), \text{Inverse homogenous}$$

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix}$$

Perspective

$$\begin{bmatrix} C_{h1} \\ C_{h2} \\ C_{h3} \\ C_{h4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix} \begin{bmatrix} kX \\ kY \\ kZ \\ k \end{bmatrix}$$

$$x = \frac{C_{h1}}{C_{h4}} = \frac{kX}{k - \frac{kZ}{f}} = \frac{fX}{f - Z}$$

$$y = \frac{C_{h2}}{C_{h4}} = \frac{kY}{k - \frac{kZ}{f}} = \frac{fY}{f - Z}$$

$$\begin{bmatrix} C_{h1} \\ C_{h2} \\ C_{h3} \\ C_{h4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix} \begin{bmatrix} kX \\ kY \\ kZ \\ k \end{bmatrix} = \begin{bmatrix} kX \\ kY \\ kZ \\ k - \frac{kZ}{f} \end{bmatrix}$$

Camera Model

- Camera is at the origin of the world coordinates first
- Then translated by some amount(G),
- Then rotated around Z axis in counter clockwise direction,
- Then rotated again around X in counter clockwise direction, and
- Then translated by C.

$$C_h = PCR^{X}_{-\phi} R^{Z}_{-\theta} G W_h$$

Since we are moving the camera instead of object we need to use inverse transformations

Camera Model

$$C_h = PCR^X_{-\phi}R^Z_{-\theta}GW_h$$

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix}, R^Z_{-\theta} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R^X_{-\phi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & \sin\phi & 0 \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 1 & 0 & 0 & -X_0 \\ 0 & 1 & 0 & -Y_0 \\ 0 & 0 & 1 & -Z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & -r_1 \\ 0 & 1 & 0 & -r_2 \\ 0 & 0 & 1 & -r_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Camera Model

$$C_h = PCR^X_{-\phi} R^Z_{-\theta} G W_h$$

$$x = f \frac{(X - X_0)\cos\theta + (Y - Y_0)\sin\theta - r_1}{-(X - X_0)\sin\theta\sin\phi + (Y - Y_0)\cos\theta\sin\phi - (Z - Z_0)\cos\phi + r_3 + f}$$

$$y = f \frac{(X - X_0)\sin\theta\cos\phi + (Y - Y_0)\cos\theta\cos\phi + (Z - Z_0)\sin\phi - r_2}{-(X - X_0)\sin\theta\sin\phi + (Y - Y_0)\cos\theta\sin\phi - (Z - Z_0)\cos\phi + r_3 + f}$$

Camera Model

$$C_h = PCR^X_{-\phi} R^Z_{-\theta} G W_h$$

$$x = \frac{C_{h1}}{C_{h4}}$$

$$y = \frac{C_{h2}}{C_{h4}}$$

$$C_h = A W_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$Ch_1 = a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4 x$$

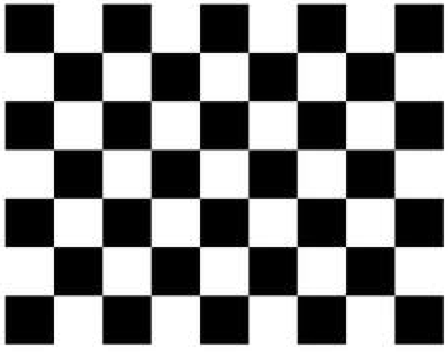
$$Ch_2 = a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4 y$$

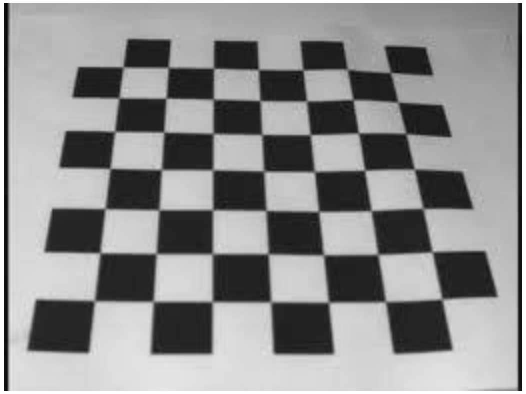
$$Ch_4 = a_{41}X + a_{42}Y + a_{43}Z + a_{44}$$

Ch_3 is not needed, we have 12 unknowns.

Camera Model

- How to determine camera matrix?
- Select some known 3D points (X,Y,Z), and find their corresponding image points (x,y).
- Solve for camera matrix elements using least squares fit.





Camera Model

$$C_h = PCR^X_{-\phi} R^Z_{-\theta} G W_h$$

$$x = \frac{C_{h1}}{C_{h4}}$$

$$y = \frac{C_{h2}}{C_{h4}}$$

$$C_h = AW_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$Ch_1 = a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4 x$$

$$Ch_2 = a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4 y$$

$$Ch_4 = a_{41}X + a_{42}Y + a_{43}Z + a_{44}$$

Ch_3 is not needed, we have 12 unknowns.

Camera Model

$$Ch_1 = a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4 x$$

$$Ch_2 = a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4 y$$

$$Ch_4 = a_{41}X + a_{42}Y + a_{43}Z + a_{44}$$

$$a_{11}X + a_{12}Y + a_{13}Z + a_{14} - a_{41}Xx - a_{42}Yx - a_{43}Zx - a_{44}x = 0$$

$$a_{21}X + a_{22}Y + a_{23}Z + a_{24} - a_{41}Xy - a_{42}Yy - a_{43}Zy - a_{44}y = 0$$

Camera Model

$$a_{11}X + a_{12}Y + a_{13}Z + a_{14} - a_{41}Xx - a_{42}Yx - a_{43}Zx - a_{44}x = 0$$

$$a_{21}X + a_{22}Y + a_{23}Z + a_{24} - a_{41}Xy - a_{42}Yy - a_{43}Zy - a_{44}y = 0$$

One point

$$a_{11}X_1 + a_{12}Y_1 + a_{13}Z_1 + a_{14} - a_{41}X_1x_1 - a_{42}Y_1x_1 - a_{43}Z_1x_1 - a_{44}x_1 = 0$$

$$a_{11}X_2 + a_{12}Y_2 + a_{13}Z_2 + a_{14} - a_{41}X_2x_2 - a_{42}Y_2x_2 - a_{43}Z_2x_2 - a_{44}x_2 = 0$$

$$\vdots$$

$$a_{11}X_n + a_{12}Y_n + a_{13}Z_n + a_{14} - a_{41}X_nx_n - a_{42}Y_nx_n - a_{43}Z_nx_n - a_{44}x_n = 0$$

$$a_{21}X_1 + a_{22}Y_1 + a_{23}Z_1 + a_{24} - a_{41}X_1y_1 - a_{42}Y_1y_1 - a_{43}Z_1y_1 - a_{44}y_1 = 0$$

$$a_{21}X_2 + a_{22}Y_2 + a_{23}Z_2 + a_{24} - a_{41}X_2y_2 - a_{42}Y_2y_2 - a_{43}Z_2y_2 - a_{44}y_2 = 0$$

$$\vdots$$

$$a_{21}X_n + a_{22}Y_n + a_{23}Z_n + a_{24} - a_{41}X_ny_n - a_{42}Y_ny_n - a_{43}Z_ny_n - a_{44}y_n = 0$$

n points
2n equations,
12 unknowns

Camera Model

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\ X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -x_2X_2 & -x_2X_2 & -x_2X_2 & -x_2 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_nZ_n & -x_n \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1X_1 & -y_1Z_1 & -y_1 \\ 0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -y_2X_2 & -y_2X_2 & -y_2Z_2 & -y_2 \\ & & & & & & \vdots & & & & & \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -y_nX_n & -y_nX_n & -y_nX_n & -y_n \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{41} \\ a_{42} \\ a_{43} \\ a_{44} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

This is a homogenous system, no unique solution $CP = 0$

Select $a_{44} = 1$

Camera Model

$$
\begin{bmatrix}
X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 \\
X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -x_2X_2 & -x_2X_2 & -x_2X_2 \\
 & & & & & & \vdots & & & & \\
X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_nZ_n \\
0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1X_1 & -y_1Z_1 \\
0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -y_2X_2 & -y_2X_2 & -y_2Z_2 \\
 & & & & & & \vdots & & & & \\
0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -y_nX_n & -y_nX_n & -y_nX_n
\end{bmatrix}
\begin{bmatrix}
a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{41} \\ a_{42} \\ a_{43} \\ \\
\end{bmatrix}
=
\begin{bmatrix}
x_1 \\ x_2 \\ \\ \\ \\ x_n \\ y_1 \\ y_2 \\ \\ \\ \\ y_n
\end{bmatrix}
$$

$$DQ = R$$

$$D^T DQ = D^T R$$

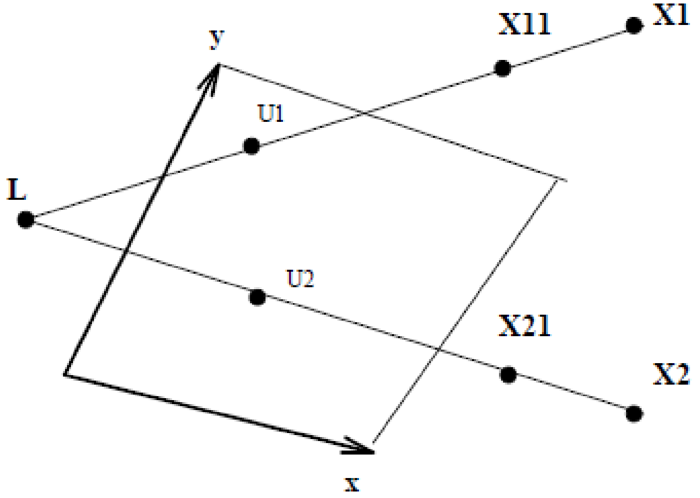
$$Q = (D^T D)^{-1} D^T R$$

Pseudo inverse

Least squares fit

Finding Camera Location

- Take one 3D point X_1 and find its image homogenous coordinates.
- Set the third component of homogenous coordinates to zero, find corresponding World coordinates of that point, X_{11}
- Connect X_1 and X_{11} to get a line in 3D.
- Repeat this for another 3D point X_2 and find another line
- Two lines will intersect at the location of camera.



Camera Location

$$C_h = AW_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$U_1 = AX_1$$

$$U_1' = \begin{pmatrix} Ch_1 & Ch_2 & 0 & Ch_4 \end{pmatrix}$$

$$X_{11} = A^{-1}U_1'$$

$$U_2 = AX_2$$

$$U_2' = ((Ch_1 \quad Ch_2 \quad 0 \quad Ch_4)$$

$$X_{22} = A^{-1}U_2'$$

Camera Orientation

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = A \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$x = \frac{C_{h1}}{C_{h4}}$$

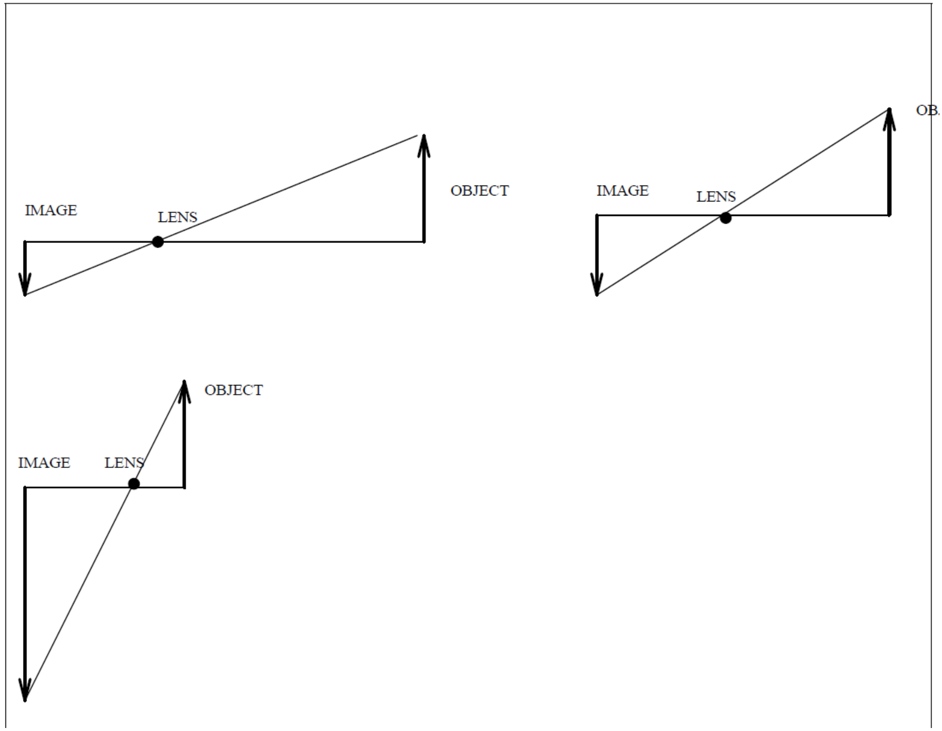
$$y = \frac{C_{h2}}{C_{h4}}$$

- Only time the image will be formed at infinity if $Ch_4=0$.

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ 0 \end{bmatrix} = A \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$a_{41}X + a_{42}Y + a_{43}Z + a_{44} = 0$$

- This is equation of a plane, going through the lens, which is parallel to image plane.



Application

$$M = \begin{bmatrix} .17237 & -.15879 & .01879 & 274.943 \\ .131132 & .112747 & .2914 & 258.686 \\ .000346 & .0003 & .00006 & 1 \end{bmatrix}$$

Camera location: intersection of California and Mason streets, at an elevation of 435 feet above sea level. The camera was oriented at an angle of 8^0 above the horizon. $fs_x=495$, $fs_y=560$.

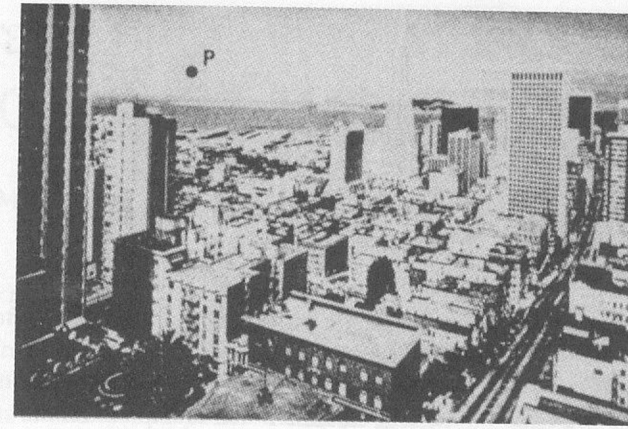


FIGURE 8 PHOTOGRAPH OF SAN FRANCISCO

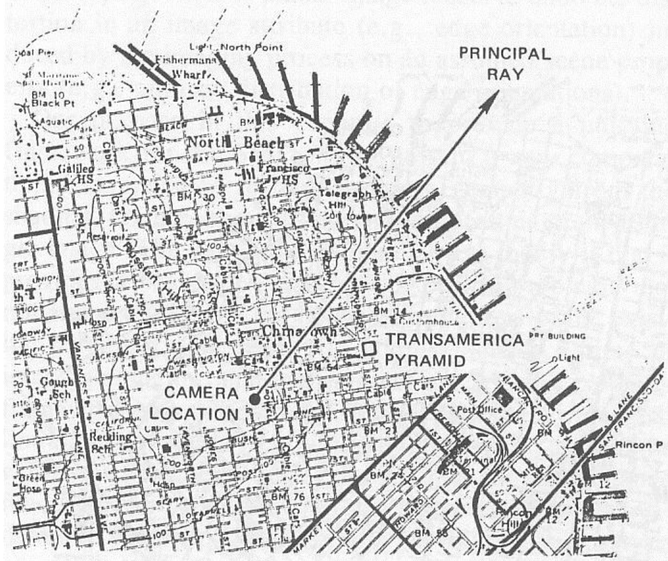


FIGURE 9 MAP OF SAN FRANCISCO

Application

$$M = \begin{bmatrix} -.175451 & -.10520 & .00435 & 297.83 \\ .02698 & -.09635 & .2303 & 249.574 \\ .00015 & -.00016 & .00001 & 1.0 \end{bmatrix}$$

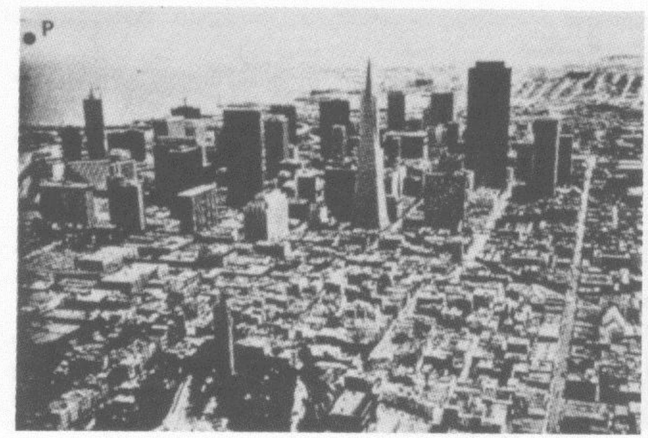


FIGURE 10 ANOTHER PHOTOGRAPH OF SAN FRANCISCO

Camera location: at an elevation of 1200 feet above sea level. The camera was oriented at an angle of 4^0 above the horizon. $fs_x=876$, $fs_y=999$.

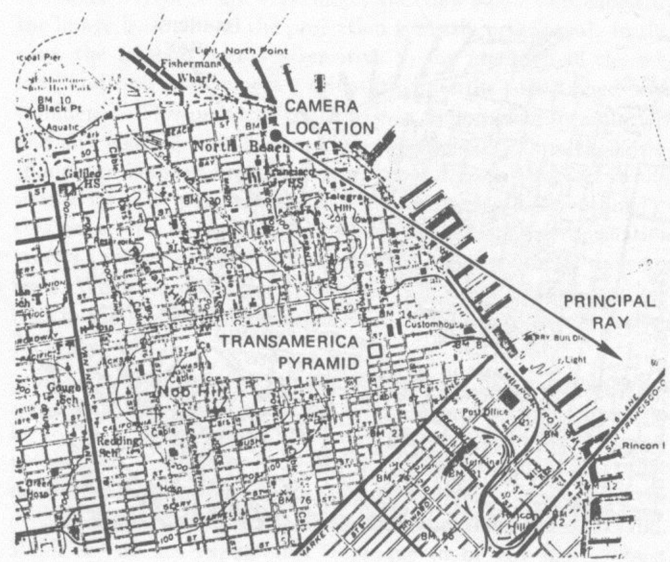


FIGURE 11 MAP OF SAN FRANCISCO

Recovering the camera parameters from a transformation matrix
TM **Strat -** Readings in Computer Vision, 1987

Camera Parameters

- Extrinsic parameters
  - Parameters that define the location and orientation of the camera reference frame with respect to a known world reference frame
    - 3-D translation vector
    - A 3 by 3 rotation matrix
- Intrinsic parameters
  - Parameters necessary to link the pixel coordinates of an image point with the corresponding coordinates in the camera reference frame
    - Perspective projection (focal length)
    - Transformation between camera frame coordinates and pixel coordinates

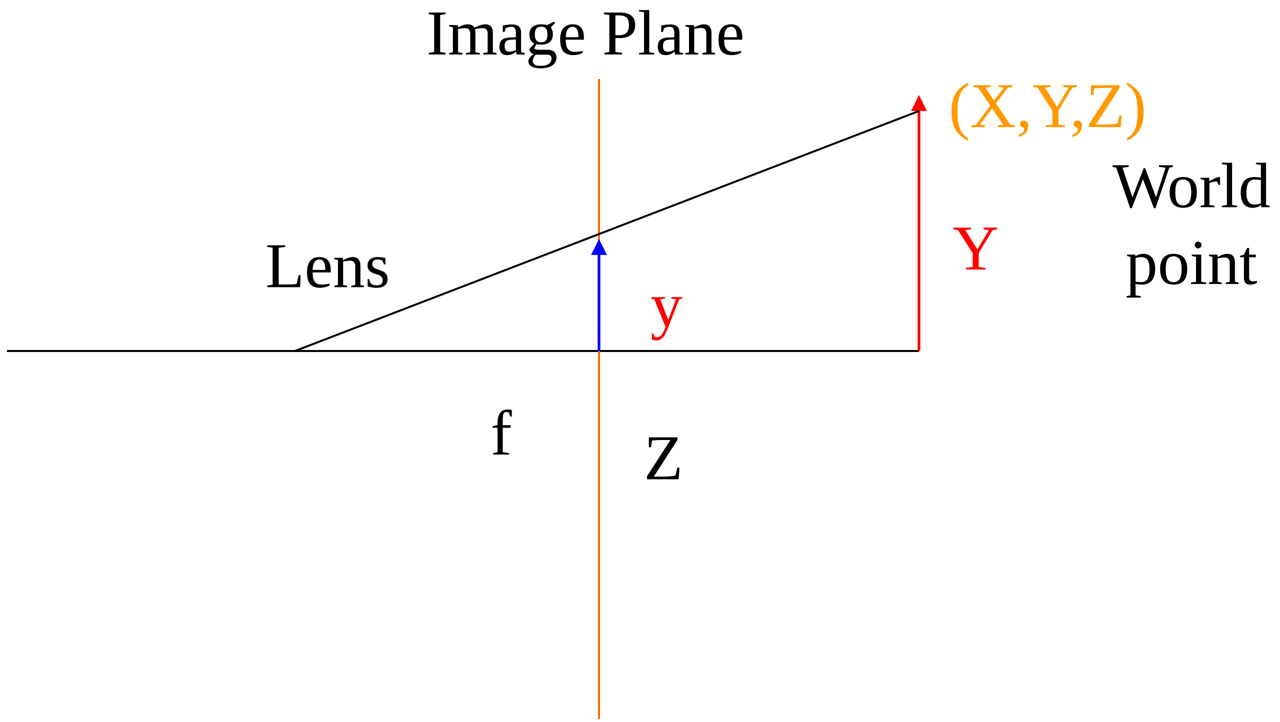
Camera Model Revisited: Rotation & Translation

$$P_c = TRP_w = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$P_c = \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$P_c = M_{ext} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Perspective Projection: Revisited



$$\frac{y}{Y} = \frac{f}{Z}$$

$$y = \frac{fY}{Z} \qquad x = \frac{fX}{Z}$$

Origin at the lens
Image plane in front of the lens

Camera Model Revisited: Perspective

$$C_h = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$x = \frac{fX}{Z}$$

$$y = \frac{fY}{Z}$$

Origin at the lens
Image plane in front of the lens

Camera Model Revisited: Image and Camera coordinates

$$x = -(x_{im} - o_x)s_x$$

$$y = -(y_{im} - o_y)s_y$$

$$x_{im} = -\frac{x}{s_x} + o_x$$

$$y_{im} = -\frac{y}{s_y} + o_y$$

$$\begin{bmatrix} x_{im} \\ y_{im} \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{s_x} & 0 & o_x \\ 0 & -\frac{1}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(x_{im}, y_{im}) image coordinates

(x, y) camera coordinates

(o_x, o_y) image center (in pixels)

(s_x, s_y) effective size of pixels (in millimeters) in the horizontal and vertical directions.

Camera Model Revisited

$$C_h = C'P'T'R'W_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{s_x} & 0 & o_x \\ 0 & -\frac{1}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M_{\text{int}} M_{ext} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Camera Model Revisited

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} r_{11} + r_{31} o_x & -\frac{f}{s_x} r_{12} + r_{32} o_x & -\frac{f}{s_x} r_{13} + r_{33} o_x & -\frac{f}{s_x} T_x + T_z o_x \\ -\frac{f}{s_y} r_{21} + r_{31} o_y & -\frac{f}{s_y} r_{22} + r_{32} o_y & -\frac{f}{s_y} r_{23} + r_{33} o_y & -\frac{f}{s_y} T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Camera Model Revisited

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} r_{11} + r_{31} o_x & -\frac{f}{s_x} r_{12} + r_{32} o_x & -\frac{f}{s_x} r_{13} + r_{33} o_x & -\frac{f}{s_x} T_x + T_z o_x \\ -\frac{f}{s_y} r_{21} + r_{31} o_y & -\frac{f}{s_y} r_{22} + r_{32} o_y & -\frac{f}{s_y} r_{23} + r_{33} o_y & -\frac{f}{s_y} T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

f_x effective focal length expressed in effective horizontal pixel size

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Computing Camera Parameters

- Using known 3-D points and corresponding image points, estimate camera matrix employing pseudo inverse method of section 1.6 (Fundamental of Computer Vision).
- Compute camera parameters by relating camera matrix with estimated camera matrix.
  - Extrinsic
    - Translation
    - Rotation
  - Intrinsic
    - Horizontal f_x and f_y vertical focal lengths
    - Translation o_x and o_y

Comparison

$$\hat{M} = \begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix}$$

$$M = \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

Computing Camera Parameters

estimated

$$\hat{M} = \gamma M$$

Since M is defined up to a scale factor

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$\sqrt{\hat{m}_{31}^2 + \hat{m}_{32}^2 + \hat{m}_{33}^2} = |\gamma| \sqrt{r_{31}^2 + r_{32}^2 + r_{33}^2} = |\gamma|$$

Because rotation matrix is orthonormal

Divide each entry of $\hat{M}$ by $|\gamma|$.

Computing Camera Parameters

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

- Compute T_z and third row r_{3i} *(i=1,2,3)*
- Compute o_x and o_y
- Compute f_x and f_y
- Compute r_{1i} and r_{2i} *i=1,2,3*
- Computer T_x and T_y

Computing Camera Parameters: estimating third row of rotation matrix and translation in depth

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$T_z = \sigma \hat{m}_{34}, \quad \sigma = \pm 1$$

$$r_{3i} = \sigma \hat{m}_{3i}, \qquad i = 1,2,3$$

Since we can determine T_z*>0 (origin of world reference is in front)*

Or T_z*<0(origin of world reference is in back)*

we can determine sign.

Computing Camera Parameters

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

Let

$$q_1 = \begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} \end{bmatrix}$$
$$q_2 = \begin{bmatrix} \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} \end{bmatrix}$$
$$q_3 = \begin{bmatrix} \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} \end{bmatrix}$$

Computing Camera Parameters: origin of image

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$q_1^T q_3 = \hat{m}_{11}\hat{m}_{31} + \hat{m}_{12}\hat{m}_{32} + \hat{m}_{13}\hat{m}_{33}$$

$$\begin{aligned} \hat{m}_{11}\hat{m}_{31} + \hat{m}_{12}\hat{m}_{32} + \hat{m}_{13}\hat{m}_{33} = & \left(-f_x r_{11} + r_{31} o_x \quad -f_x r_{12} + r_{32} o_x \quad -f_x r_{13} + r_{33} o_x\right) . \left(r_{31} \quad r_{32} \quad r_{33}\right) \\ & = \left(-f_x r_{11} \quad -f_x r_{12} \quad -f_x r_{13}\right) . \left(r_{31} \quad r_{32} \quad r_{33}\right) + \\ & \left(r_{31} o_x \quad r_{32} o_x \quad r_{32} o_x\right) . \left(r_{31} \quad r_{32} \quad r_{33}\right) \\ & = \left(r_{31} o_x \quad r_{32} o_x \quad r_{32} o_x\right) . \left(r_{31} \quad r_{32} \quad r_{33}\right) \\ & = (r_{31}{}^2 o_x + r_{32}{}^2 o_x + r_{33}{}^2 o_x) \\ & = o_x (r_{31}{}^2 + r_{32}{}^2 + r_{33}{}^2) \\ q_1^T q_3 \quad & = o_x \end{aligned}$$

Therefore:

$$o_x = q_1^T q_3$$

$$o_y = q_2^T q_3$$

Computing Camera Parameters: vertical and horizontal focal lengths

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$q_1^T q_1 = \hat{m}_{11}\hat{m}_{11} + \hat{m}_{12}\hat{m}_{12} + \hat{m}_{13}\hat{m}_{13}$$

$$\hat{m}_{11}\hat{m}_{11} + \hat{m}_{12}\hat{m}_{12} + \hat{m}_{13}\hat{m}_{13} =$$

$$(-f_x r_{11} + r_{31} o_x \quad -f_x r_{12} + r_{32} o_x \quad -f_x r_{13} + r_{33} o_x).(-f_x r_{11} + r_{31} o_x \quad -f_x r_{12} + r_{32} o_x \quad -f_x r_{13} + r_{33} o_x)$$

$$= (-f_x r_{11} + r_{31} o_x)^2 + (-f_x r_{12} + r_{32} o_x)^2 + (-f_x r_{13} + r_{33} o_x)^2$$

$$= (f_x{}^2 r_{11}{}^2 + r_{31}{}^2 o_x{}^2) + (f_x{}^2 r_{12}{}^2 + r_{32}{}^2 o_x{}^2) + (f_x{}^2 r_{13}{}^2 + r_{33}{}^2 o_x{}^2)$$

$$= f_x{}^2 + o_x{}^2$$

$$q_1^T q_1 = f_x{}^2 + o_x{}^2$$

$$\sqrt{q_1^T q_1 - o_x^2} = f_x$$

Therefore:

$$f_x = \sqrt{q_1^T q_1 - o_x^2}$$

$$f_y = \sqrt{q_2^T q_2 - o_y^2}$$

Computing Camera Parameters: remaining rotation and translation parameters

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$0_x r_{31} + f_x r_{11} - r_{31} 0_x = \sigma(0_x \hat{m}_{31} - \hat{m}_{11}),$$

$$r_{11} = \sigma(0_x \hat{m}_{31} - \hat{m}_{11}) / f_x,$$

$$r_{1i} = \sigma(0_x \hat{m}_{3i} - \hat{m}_{1i}) / f_x, \quad i = 1,2,3$$

$$r_{2i} = \sigma(0_y \hat{m}_{3i} - \hat{m}_{2i}) / f_y, \quad i = 1,2,3$$

$$T_x = \sigma(0_x \hat{m}_{34} - \hat{m}_{14}) / f_x$$

$$T_y = \sigma(0_y \hat{m}_{34} - \hat{m}_{24}) / f_y$$

Reading Material

- Chapter 1, Fundamental Of Computer Vision, Mubarak Shah
- Chapter 6, Introductory Techniques, E. Trucco and A. Verri, Prentice Hall, 1998.
- **Recovering the camera parameters from a transformation matrix,** TM **Strat** - Readings in Computer Vision, 1987