CAP 5415 Computer Vision Fall 2011

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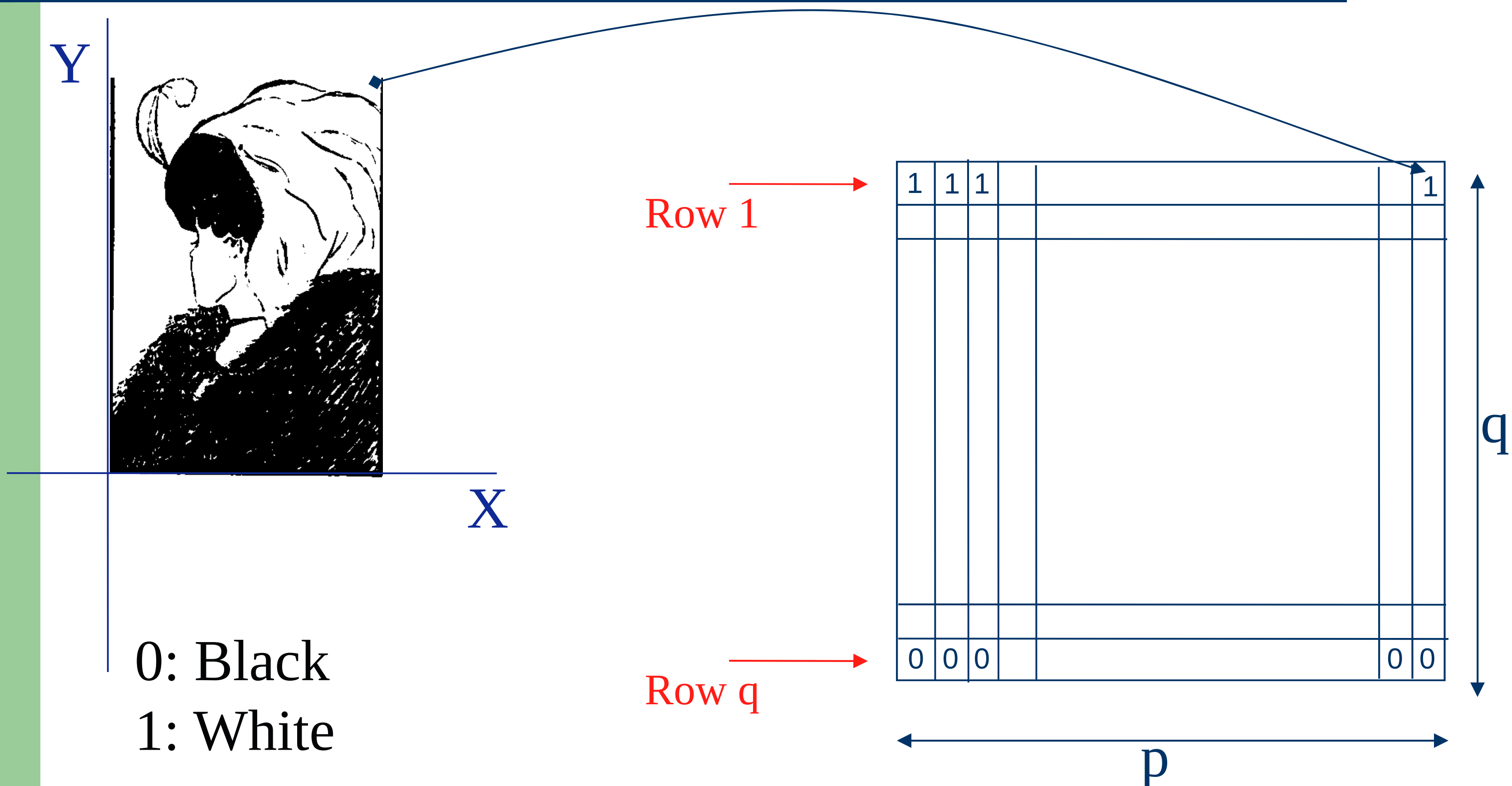
Filtering

Lecture-2

General

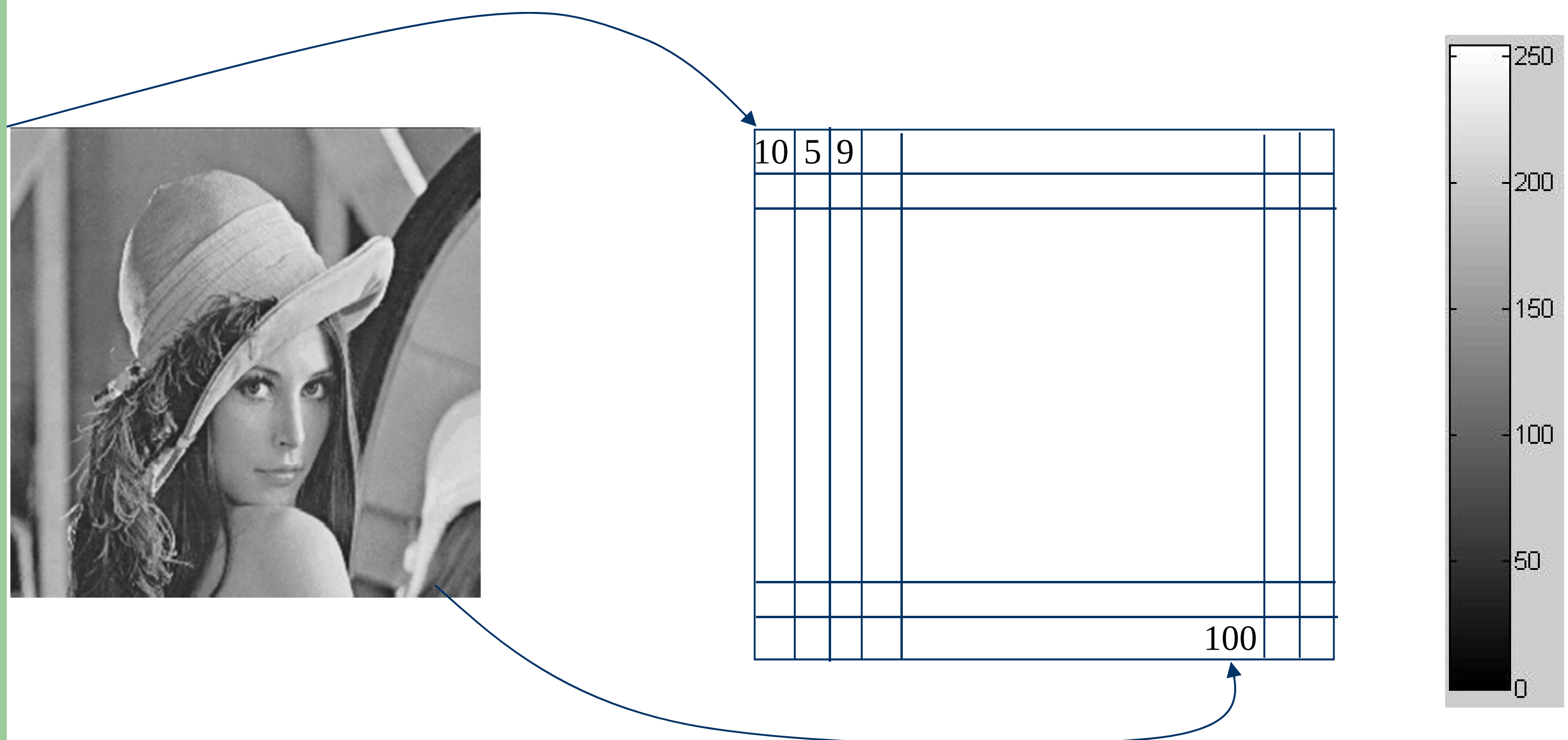
- Binary
- Gray Scale
- Color

Binary Images



0: Black
1: White

Gray Level Image



Gray Scale Image

Color Image
Red, Green, Blue Channels

Image Histogram

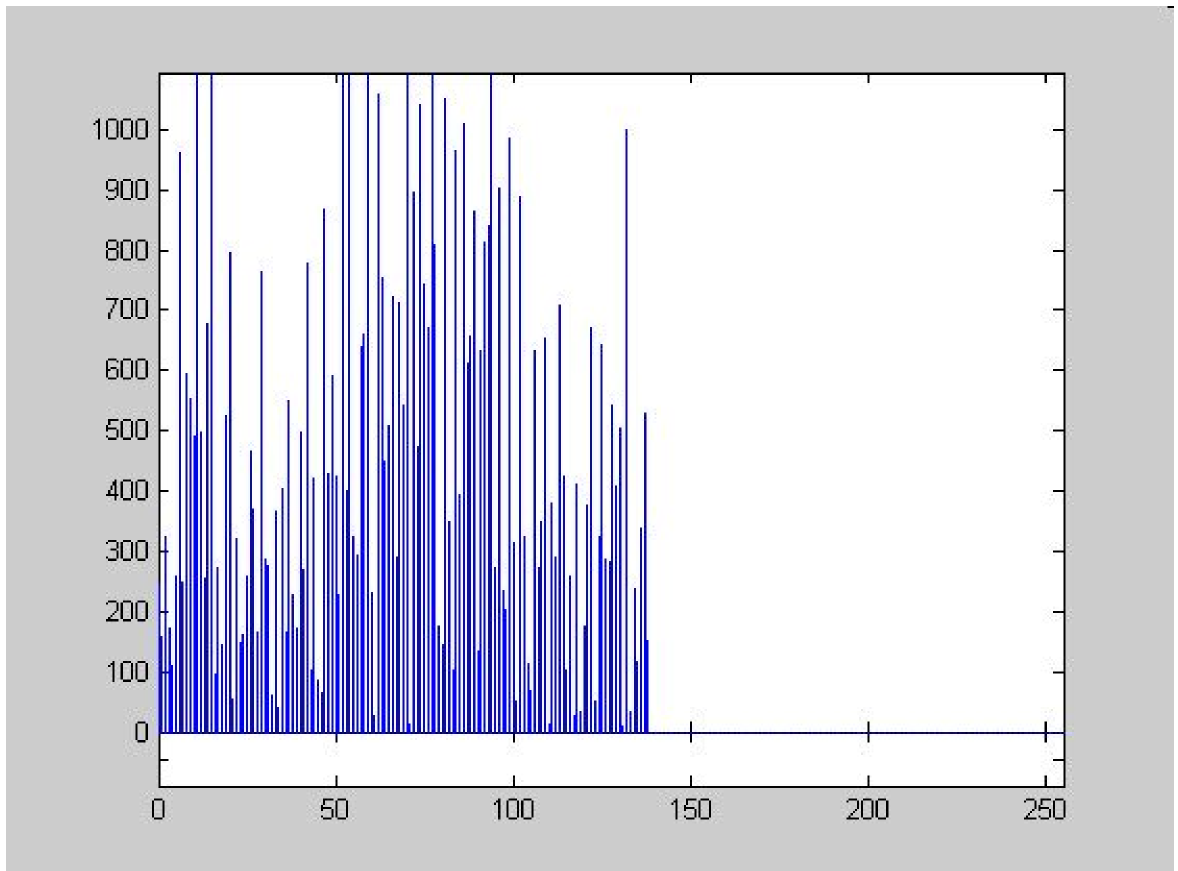


Image Noise

- Light Variations
- Camera Electronics
- Surface Reflectance
- Lens

Image Noise

- $I(x,y)$: the true pixel values
- $n(x,y)$: the noise at pixel (x,y)

$$\hat{I}(x,y) = I(x,y) + n(x,y)$$

➔

Gaussian Noise

$$n(x,y)=e^{\frac{-n^2}{2\sigma^2}}$$

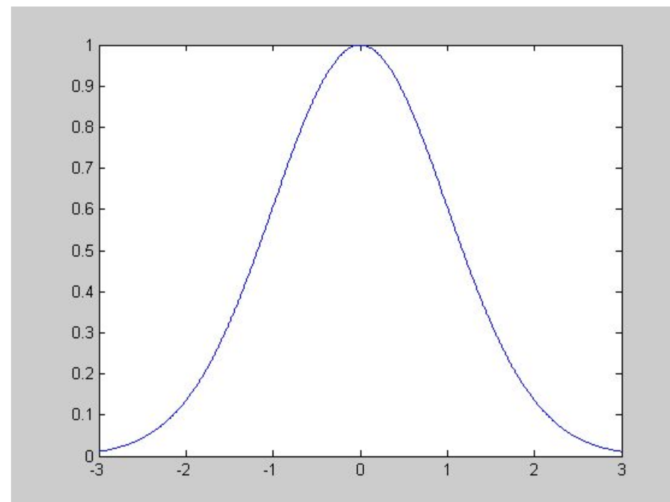


Image Derivatives & Averages

Definitions

- Derivative: Rate of change
  - *Speed* is a rate of change of a *distance*
  - *Acceleration* is a rate of change of *speed*
- Average (Mean)
  - Dividing the sum of N values by N

Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x) = f_x$$

$$v = \frac{ds}{dt}$$ speed

$$a = \frac{dv}{dt}$$ acceleration

Examples

$$y = x^2 + x^4$$

$$\frac{dy}{dx} = 2x + 4x^3$$

$$y = \sin x + e^{-x}$$

$$\frac{dy}{dx} = \cos x + (-1)e^{-x}$$

Discrete Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x)$$

$$\frac{df}{dx} = \frac{f(x) - f(x-1)}{1} = f'(x)$$

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$

Discrete Derivative
Finite Difference

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$

Backward difference

$$\frac{df}{dx} = f(x) - f(x+1) = f'(x)$$

Forward difference

$$\frac{df}{dx} = f(x+1) - f(x-1) = f'(x)$$

Central difference

Example

$f(x) =$	10	15	10	10	25	20	20	20
$f'(x) =$	0	5	-5	0	15	-5	0	0
$f''(x) =$	0	5	-10	5	15	20	5	0

Derivative Masks

Backward difference	[-1 1]
Forward difference	[1 -1]
Central difference	[-1 0 1]

Derivatives in 2 Dimensions

Given function $$f(x,y)$$

Gradient vector $$\nabla f(x,y) = \begin{bmatrix} \dfrac{\partial f(x,y)}{\partial x} \\ \dfrac{\partial f(x,y)}{\partial y} \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

Gradient magnitude $$\left|\nabla f(x,y)\right| = \sqrt{f_x^2 + f_y^2}$$

Gradient direction $$\theta = \tan^{-1} \frac{f_x}{f_y}$$

Derivatives of Images

Derivative masks

$$f_x \Rightarrow \frac{1}{3}\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad f_y \Rightarrow \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

$$I = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix} \qquad I_x = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Derivatives of Images

$$I = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix} \qquad I_y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Correlation

$$f \otimes h = \sum_k \sum_l f(k,l)\,h(i+k,\, j+l)$$

f = Image

f = Kernel

f

f_1	f_2	f_3
f_4	f_5	f_6
f_7	f_8	f_9

$\otimes$

h

h_1	h_2	h_3
h_4	h_5	h_6
h_7	h_8	h_9

$$\begin{aligned} f * h = {} & f_1 h_1 + f_2 h_2 + f_3 h_3 \\ & + f_4 h_4 + f_5 h_5 + f_6 h_6 \\ & + f_7 h_7 + f_8 h_8 + f_9 h_9 \end{aligned}$$

Convolution

$$f * h = \sum_k \sum_l f(k,l)\,h(i-k, j-l)$$

f = Image

h = Kernel

h

h_1	h_2	h_3
h_4	h_5	h_6
h_7	h_8	h_9

$X - flip$

h_7	h_8	h_9
h_4	h_5	h_6
h_1	h_2	h_3

$Y - flip$

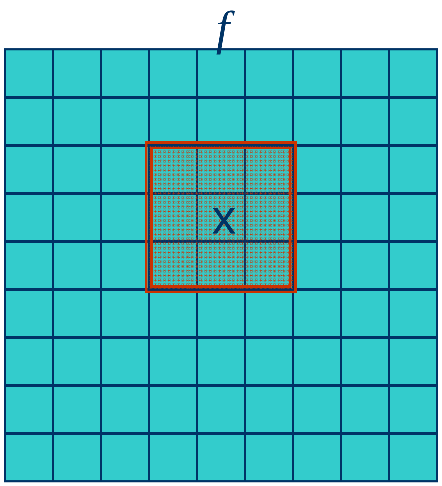
f

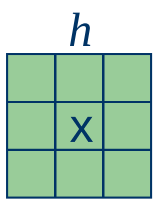
$$\begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix} \otimes \begin{bmatrix} h_9 & h_8 & h_7 \\ h_6 & h_5 & h_4 \\ h_3 & h_2 & h_1 \end{bmatrix} \rightarrow$$

$$\begin{aligned} f * h = {} & f_1 h_9 + f_2 h_8 + f_3 h_7 \\ & + f_4 h_6 + f_5 h_5 + f_6 h_4 \\ & + f_7 h_3 + f_8 h_2 + f_9 h_1 \end{aligned}$$

Convolution

$$
\begin{aligned}
f(x,y)*h = \quad & f(x+1,y+1)h(-1,-1) + f(x,y+1)h(0,-1) + f(x-1,y+1)h(1,-1) + \\
& f(x+1,y)h(-1,0) + f(x,y)h(0,0) + f(x-1,y)h(1,0) \\
& f(x+1,y-1)h(-1,1) + f(x,y-1)h(0,1) + f(x-1,y-1)h(1,1)
\end{aligned}
$$





$$
f*h = \sum_{i=-1}^{1}\sum_{j=-1}^{1} f(x-i,y-i)h(i,j)
$$

Coordinates

-1,0	**0,1**	**1,1**
-1,0	0,0	1,0
-1,-1	0,-1	1,-1

Averages

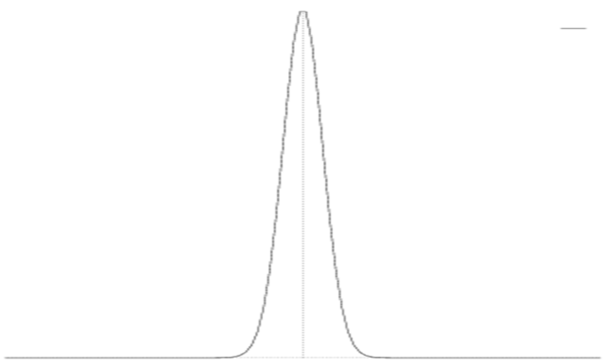
- Mean

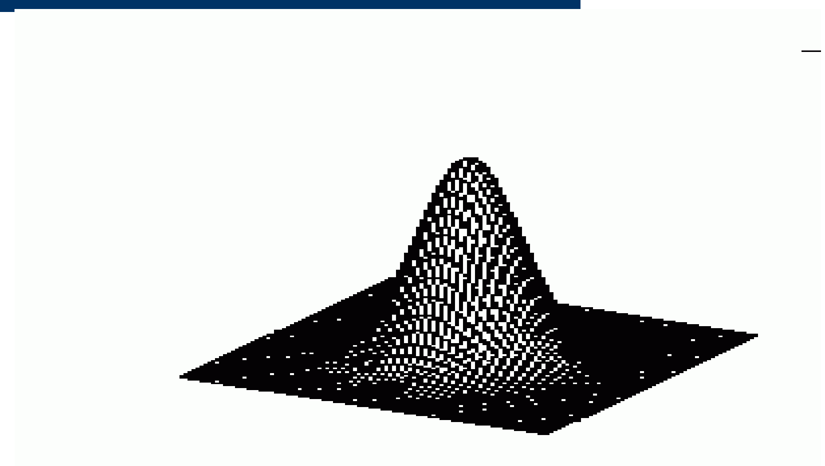
$$I = \frac{I_1 + I_2 + \ldots I_n}{n} = \frac{\sum_{i=1}^{n} I_i}{n}$$

- Weighted mean

$$I = \frac{w_1 I_1 + w_2 I_2 + \ldots + w_n I_n}{n} = \frac{\sum_{i=1}^{n} w_i I_i}{n}$$

Gaussian Filter





$$g(x) = e^{\frac{-x^2}{2o^2}}$$

$$g(x,y) = e^{\frac{-(x^2+y^2)}{2o^2}}$$

$$g(x) = [.011 \quad .13 \quad .6 \quad 1 \quad .6 \quad .13 \quad .011]$$

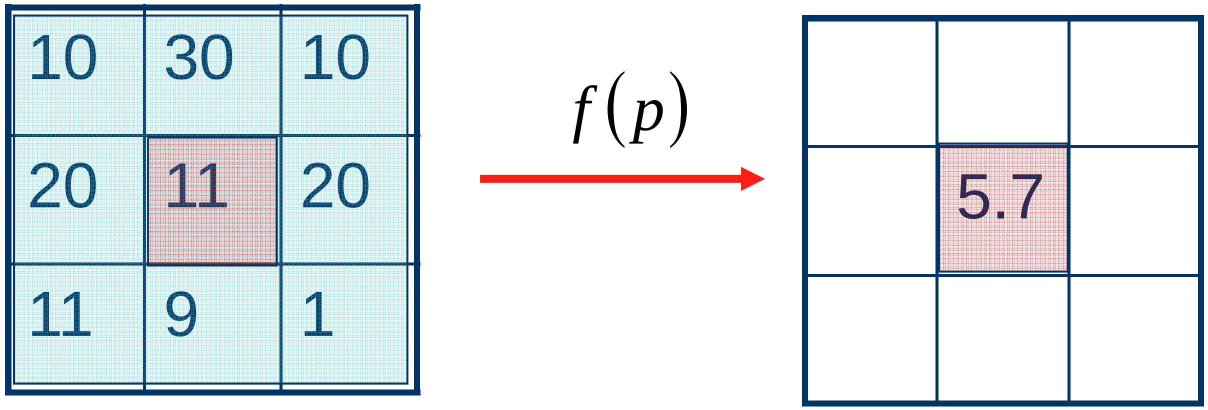
$$\sigma = 1$$

Properties of Gaussian

- Most common natural model
- Smooth function, it has infinite number of derivatives
- Fourier Transform of Gaussian is Gaussian.
- Convolution of a Gaussian with itself is a Gaussian.
- There are cells in eye that perform Gaussian filtering.

Filtering

- Modify pixels based on some function of the neighborhood



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Linear Filtering

- The output is the linear combination of the neighborhood pixels

1	3	0
2	10	2
4	1	1

Image

$\otimes$

1	0	-1
1	0.1	-1
1	0	-1

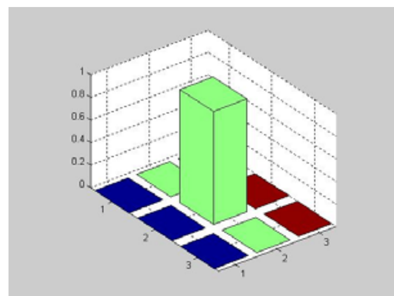
Kernel

=

	5	

Filter Output

Filtering Examples



*

0	0	0
0	1	0
0	0	0

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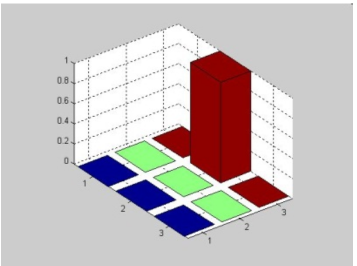
Filtering Examples

$*$

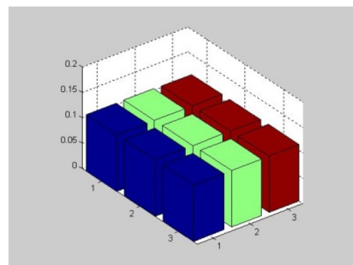


0	0	0
0	0	1
0	0	0

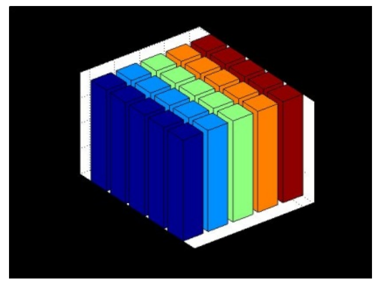
$=$

Filtering Examples

$$* \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} =$$

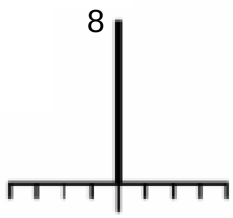


Filtering Examples

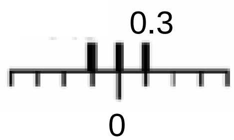


$$* \frac{1}{25} \begin{array}{|c|c|c|c|c|} \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline \end{array} =$$

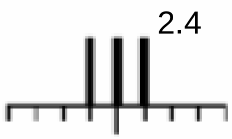
Blurring Examples



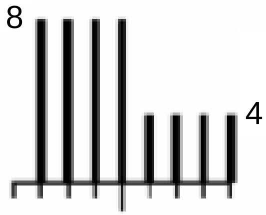
original



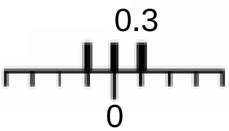
pixel offset



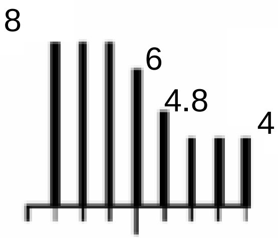
filtered



original



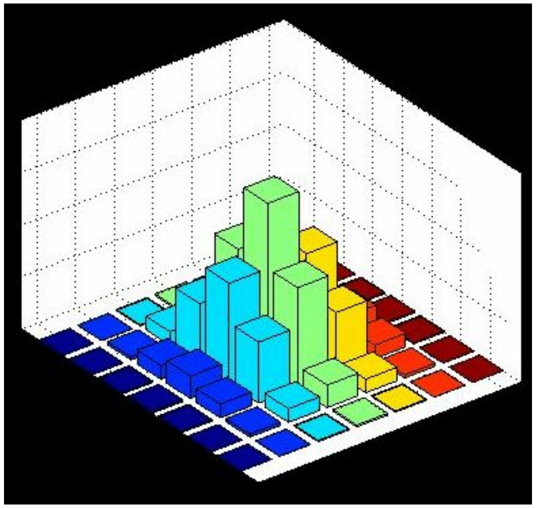
pixel offset



filtered

Filtering Gaussian

$*$



$=$

Gaussian vs. Smoothing

Gaussian Smoothing

Smoothing by Averaging

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Noise Filtering

Gaussian Noise

After Averaging

After Gaussian Smoothing

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MATLAB Functions

- conv: 1-D Convolution.
  - *C = conv(A, B)* convolves vectors A and B.

- conv2: Two dimensional convolution.
  - C = conv2(A, B) performs the 2-D convolution of matrices A and B.

MATLAB Functions

- filter2: Two-dimensional digital filter.
  - *Y = filter2(B,X)* filters the data in X with the 2-D FIR filter in the matrix B.
  - The result, Y, is computed using 2-D correlation and is the same size as X.
  - filter2 uses CONV2 to do most of the work. 2-D correlation is related to 2-D convolution by a 180 degree rotation of the filter matrix.

MATLAB Functions

- gradient: Approximate gradient.
  - *[FX,FY] = gradient(F)* returns the numerical gradient of the matrix F. FX corresponds to dF/dx, FY corresponds to dF/dy.
- mean: Average or mean value.
  - For vectors, *mean(X)* is the mean value (average) of the elements in X.

MATLAB Functions

- special: Create predefined 2-D filters
  - *H = fspecial(TYPE)* creates a two-dimensional filter H of the specified type. Possible values for TYPE are:
  - 'average' averaging filter;
  - 'gaussian' Gaussian lowpass filter
  - 'laplacian' filter approximating the 2-D Laplacian operator
  - 'log' Laplacian of Gaussian filter
  - 'prewitt' Prewitt horizontal edge-emphasizing filter
  - 'sobel' Sobel horizontal edge-emphasizing filter
  - Example: H=fspecial('gaussian',7,1) creates a 7x7 Gaussian filter with variance 1.