CAP 5415 Computer Vision Fall 2012

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Edge Detection

Lecture-3

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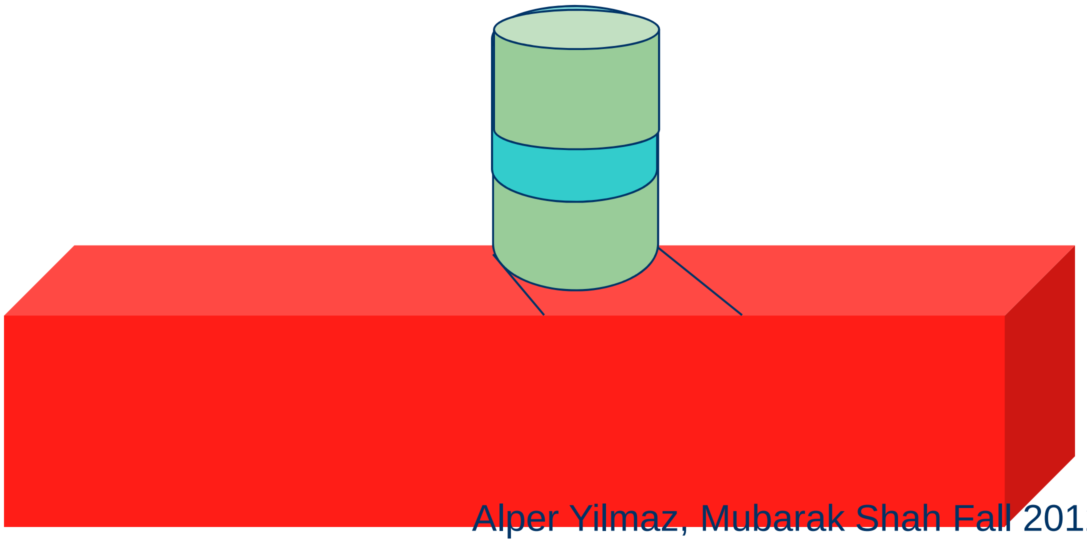
Example

An Application

- What is an object?
- How can we find it?

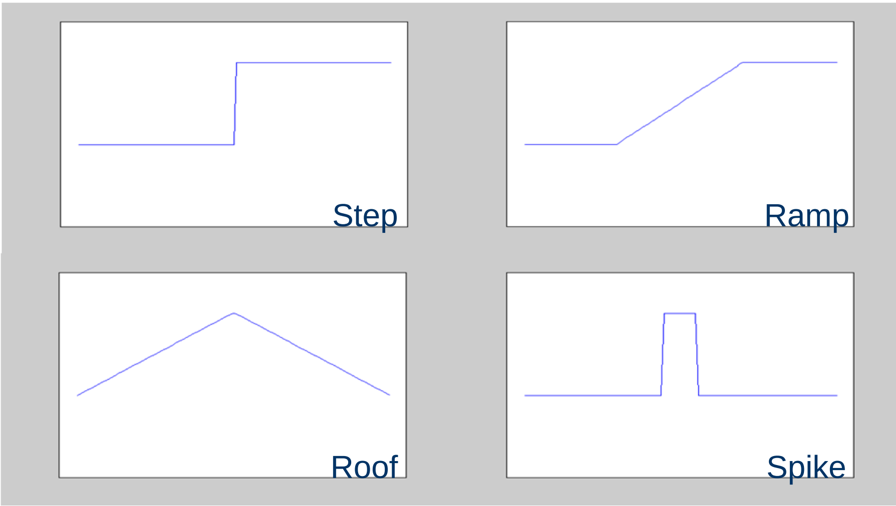
Edge Detection in Images

- At edges intensity or color changes



What is an Edge?

- Discontinuity of intensities in the image
- Edge models
  - Step
  - Roof
  - Ramp
  - Spike



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Detecting Discontinuities

- Image derivatives

$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \to 0}\left(\frac{f(x+\varepsilon)-f(x)}{\varepsilon}\right) \Longrightarrow \frac{\partial f}{\partial x} \approx \frac{f(x_{n+1})-f(x)}{\Delta x}$$

- Convolve image with derivative filters

Backward difference	[-1 1]
Forward difference	[1 -1]
Central difference	[-1 0 1]

Derivative in Two-Dimensions

- Definition

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0}\left(\frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}\right) \qquad \frac{\partial f(x,y)}{\partial y} = \lim_{\varepsilon \to 0}\left(\frac{f(x, y+\varepsilon) - f(x,y)}{\varepsilon}\right)$$

- Approximation

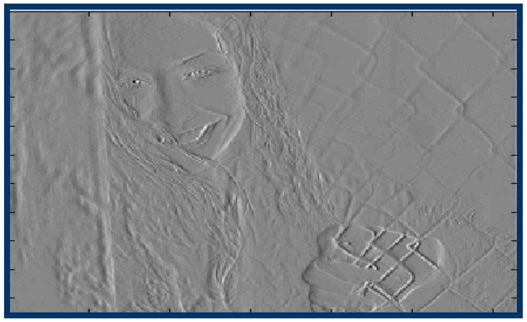
$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x_{n+1}, y_m) - f(x_n, y_m)}{\Delta x} \qquad \frac{\partial f(x,y)}{\partial y} \approx \frac{f(x_n, y_{m+1}) - f(x_n, y_m)}{\Delta x}$$

- Convolution kernels

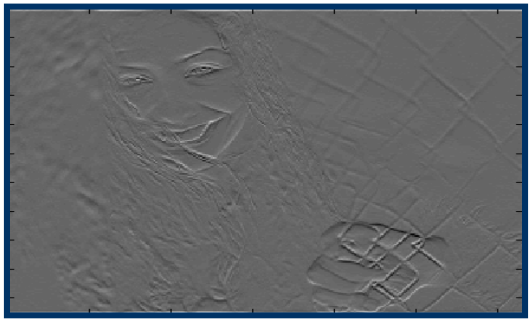
$$f_x = \begin{bmatrix} 1 & -1 \end{bmatrix} \qquad f_y = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Image Derivatives

Image I



$I_x = I * \begin{bmatrix} 1 & -1 \end{bmatrix}$



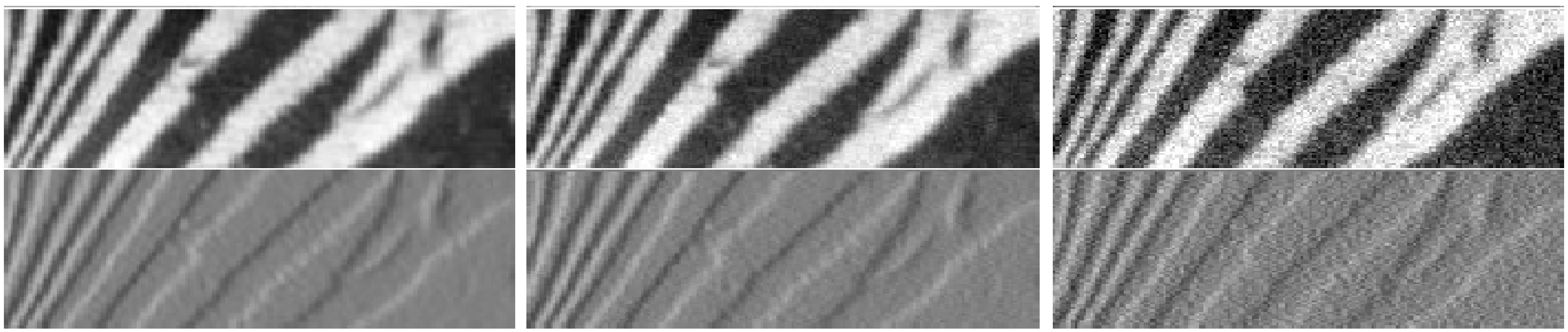
$I_y = I * \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Derivatives and Noise

- Strongly affected by noise
  - obvious reason: image noise results in pixels that look very different from their neighbors
- The larger the noise is the stronger the response

- What is to be done?
  - Neighboring pixels look alike
  - Pixel along an edge look alike
  - Image smoothing should help
    - Force pixels different to their neighbors (possibly noise) to look like neighbors

Derivatives and Noise



Increasing noise →

Zero mean additive gaussian noise

Image Smoothing

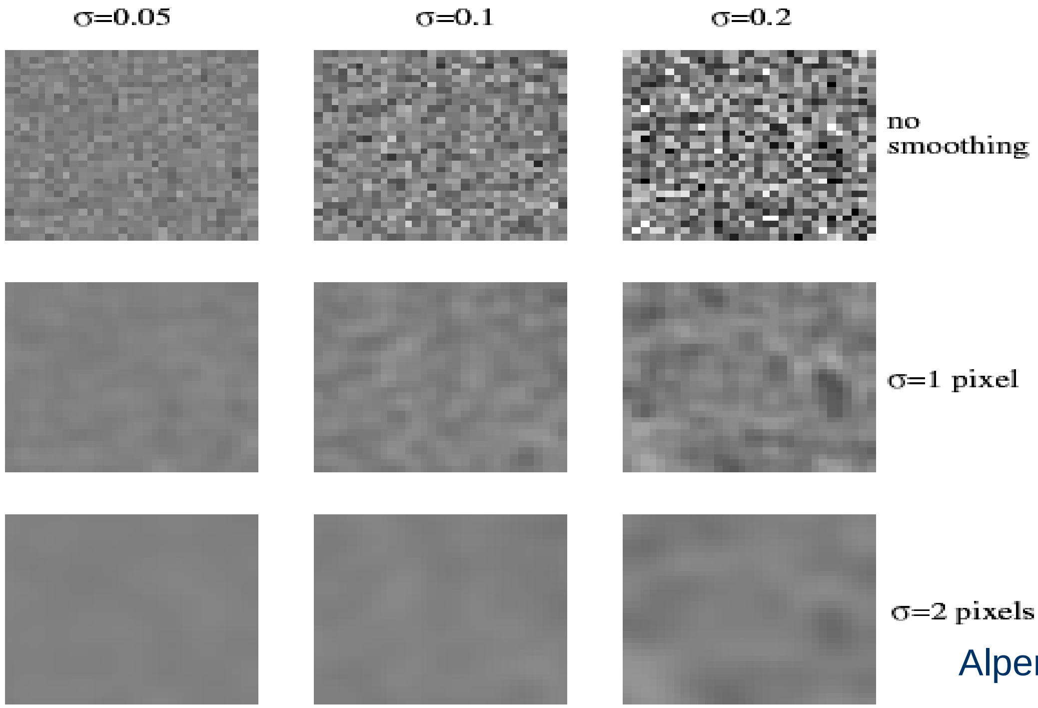
- Expect pixels to "**be like**" their neighbors
  - Relatively few reflectance changes
- Generally expect noise to be independent from pixel to pixel
  - Smoothing suppresses noise

Gaussian Smoothing

$$g(x, y) = e^{\frac{-(x^2 + y^2)}{2\sigma^2}}$$

- Scale of Gaussian σ
  - As σ increases, more pixels are involved in average
  - As σ increases, image is more blurred
  - As σ increases, noise is more effectively suppressed

Gaussian Smoothing (Examples)



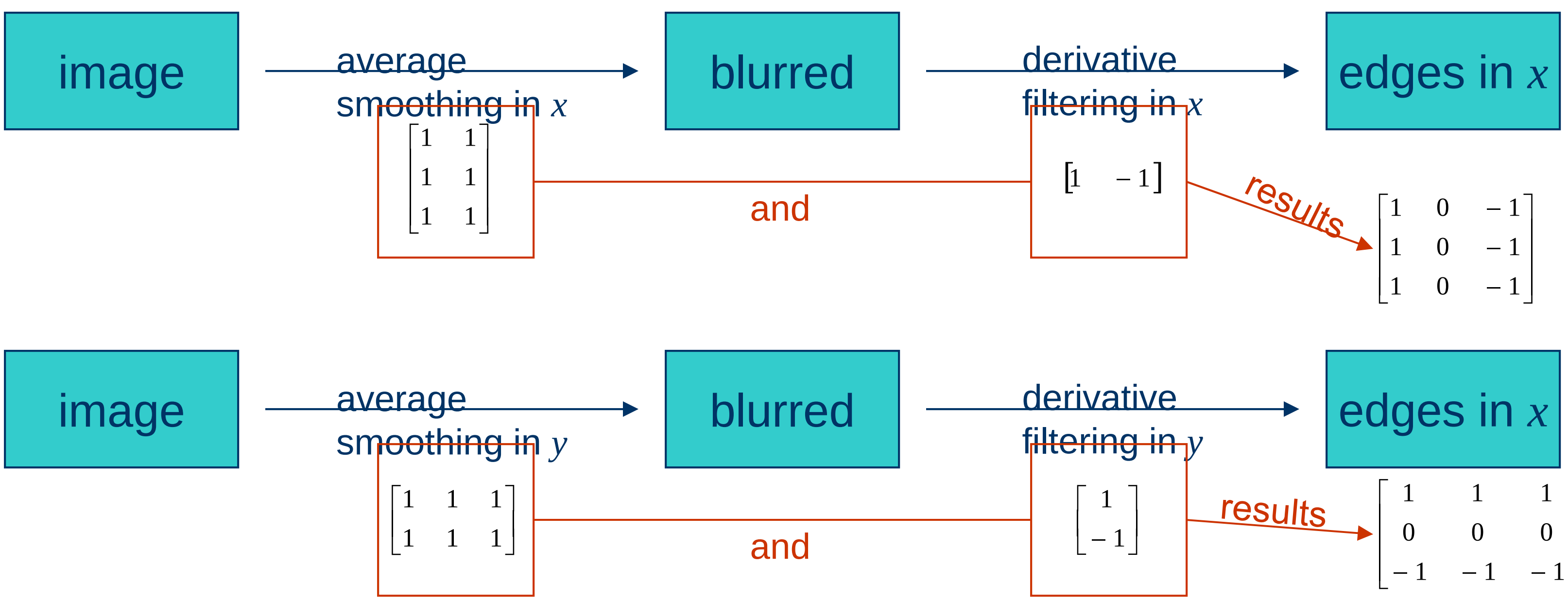
Edge Detectors

- Gradient operators
  - Prewit
  - Sobel
- Laplacian of Gaussian (Marr-Hildreth)
- Gradient of Gaussian (Canny)

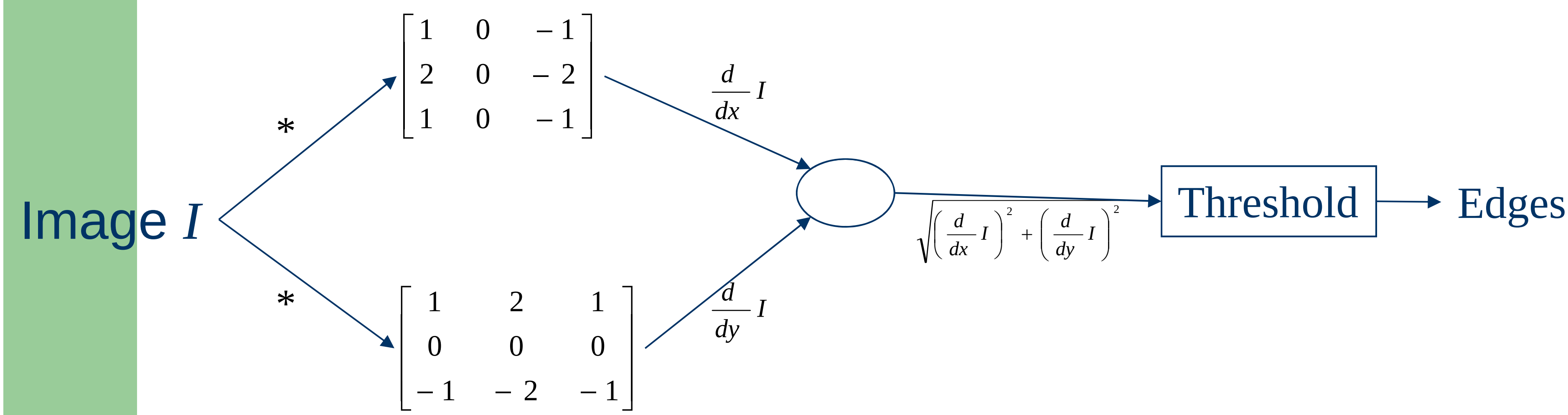
Prewitt and Sobel Edge Detector

- Compute derivatives
  - In x and y directions
- Find gradient magnitude
- Threshold gradient magnitude

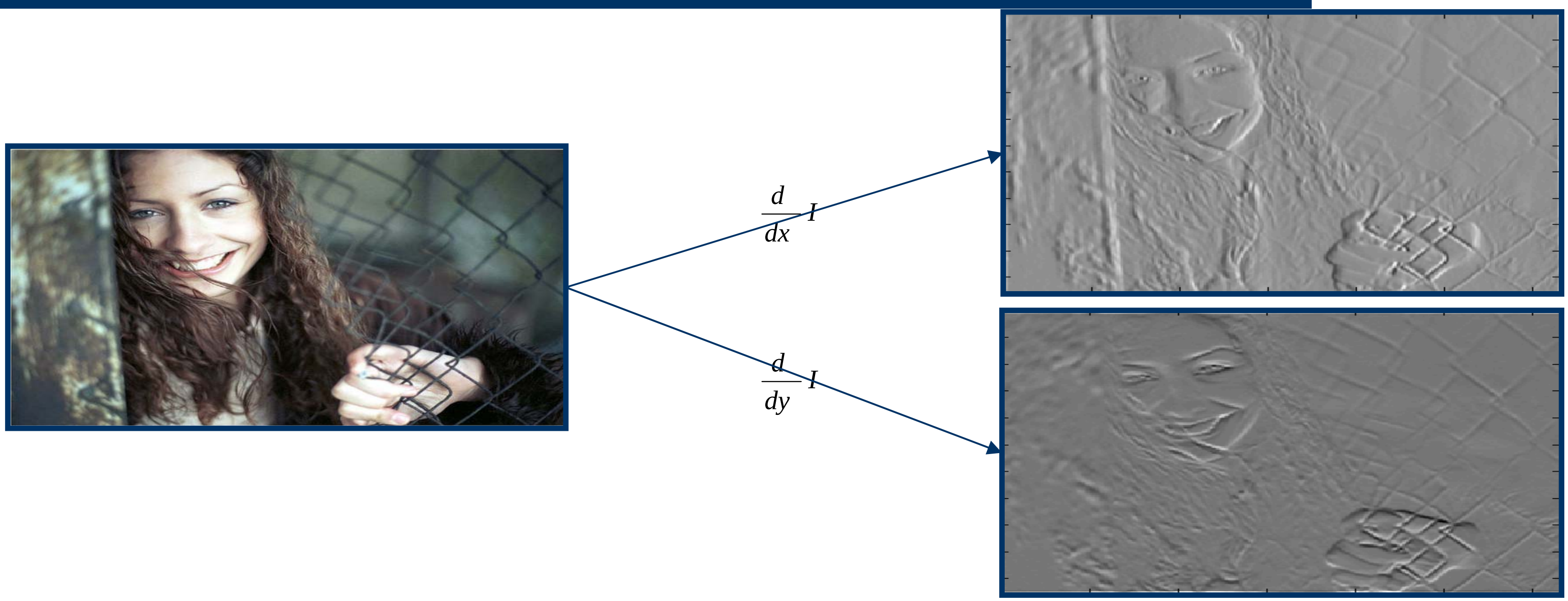
Prewitt Edge Detector



Sobel Edge Detector



Sobel Edge Detector



Sobel Edge Detector

$$\Delta = \sqrt{\left(\frac{d}{dx}I\right)^2 + \left(\frac{d}{dy}I\right)^2}$$

$$\Delta \geq Threshold = 100$$

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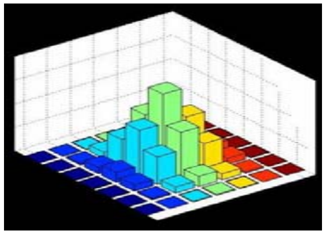
Marr Hildreth Edge Detector

- Smooth image by Gaussian filter → S
- Apply Laplacian to S
  - Used in mechanics, electromagnetics, wave theory, quantum mechanics and Laplace equation
- Find zero crossings
  - Scan along each row, record an edge point at the location of zero-crossing.
  - Repeat above step along each column

Marr Hildreth Edge Detector

- Gaussian smoothing

$$\overbrace{S}^{\text{smoothed image}} = \overbrace{g}^{\text{Gaussian filter}} * \overbrace{I}^{\text{image}} \qquad g = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



- Find Laplacian

$$\Delta^2 S = \overbrace{\frac{\partial^2}{\partial x^2} S}^{\text{second order derivative in } x} + \overbrace{\frac{\partial^2}{\partial y^2} S}^{\text{second order derivative in } y}$$

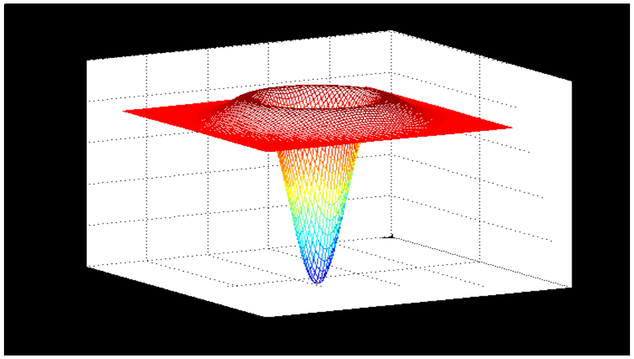
- ∇ is used for gradient (derivative)
- Δ is used for Laplacian

Marr Hildreth Edge Detector

- Deriving the Laplacian of Gaussian (LoG)

$$\Delta^2 S = \Delta^2 (g * I) = (\Delta^2 g) * I$$

$$\Delta^2 g = -\frac{1}{\sqrt{2\pi}\,\sigma^3}\left(2 - \frac{x^2 + y^2}{\sigma^2}\right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$



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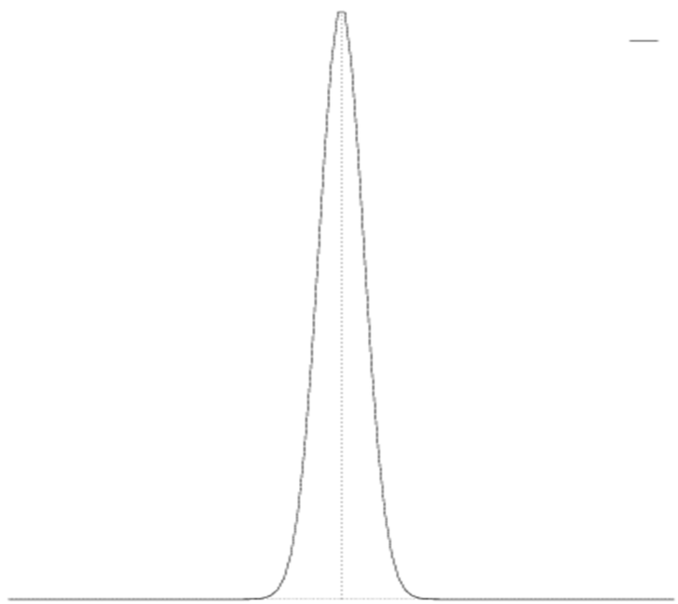
Gaussian

$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

Standard deviation

x	-3	-2	-1	0	1	2	3
g(x)	.011	.13	.6	1	.6	.13	.011



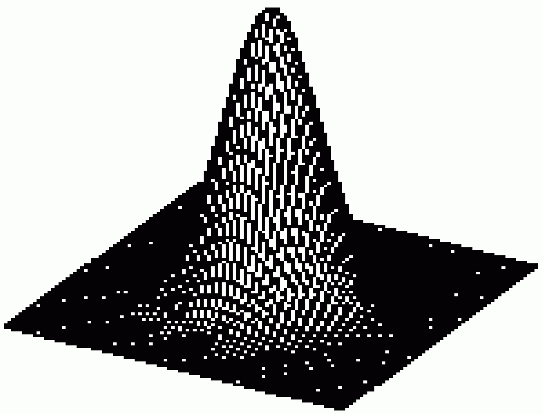
2-D Gaussian

$$g(x, y) = e^{\frac{-(x^2 + y^2)}{2\sigma^2}}$$

0	0	0	0	1	2	2	2	1	0	0	0	0
0	0	1	3	6	9	11	9	6	3	1	0	0
0	1	4	11	20	30	34	30	20	11	4	1	0
0	3	11	26	50	73	82	73	50	26	11	3	0
1	6	20	50	93	136	154	136	93	50	20	6	1
2	9	30	73	136	198	225	198	136	73	30	9	2
2	11	34	82	154	225	255	225	154	82	34	11	2
2	9	30	73	136	198	225	198	136	73	30	9	2
1	6	20	50	93	136	154	136	93	50	20	6	1
0	3	11	26	50	73	82	73	50	26	11	3	0
0	1	4	11	20	30	34	30	20	11	4	1	0
0	0	1	3	6	9	11	9	6	3	1	0	0
0	0	0	0	1	2	2	2	1	0	0	0	0

$$\sigma = 2$$

2-D Gaussian



LoG Filter

$$\Delta^2 G_\sigma = -\frac{1}{\sqrt{2\pi}\,\sigma^3}\left(2 - \frac{x^2 + y^2}{\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Y

0.0008	0.0066	0.0215	0.031	0.0215	0.0066	0.0008
0.0066	0.0438	0.0982	0.108	0.0982	0.0438	0.0066
0.0215	0.0982	0	-0.242	0	0.0982	0.0215
0.031	0.108	-0.242	-0.7979	-0.242	0.108	0.031
0.0215	0.0982	0	-0.242	0	0.0982	0.0215
0.0066	0.0438	0.0982	0.108	0.0982	0.0438	0.0066
0.0008	0.0066	0.0215	0.031	0.0215	0.0066	0.0008

X

Finding Zero Crossings

- Four cases of zero-crossings :
  - {+,-}
  - {+,0,-}
  - {-,+}
  - {-,0,+}
- Slope of zero-crossing {a, -b} is |a+b|.
- To mark an edge
  - compute slope of zero-crossing
  - Apply a threshold to slope

On the Separability of LoG

- Similar to separability of Gaussian filter
  - Two-dimensional Gaussian can be separated into 2 one-dimensional Gaussians

$$h(x,y) = I(x,y) * g(x,y)$$

n^2 multiplications

$$h(x,y) = \left(I(x,y) * g_1(x)\right) * g_2(y)$$

$2n$ multiplications

$$g(x) = e^{-\left(\frac{x^2}{2\sigma^2}\right)}$$

$$g_2 = g(y) = \begin{bmatrix} .011 \\ .13 \\ .6 \\ 1 \\ .6 \\ .13 \\ .011 \end{bmatrix}$$

$$g_1 = g(x) = [.011 \quad .13 \quad .6 \quad 1 \quad .6 \quad .13 \quad .011]$$

On the Separability of LoG

$$\Delta^2 S = \Delta^2 (g * I) = (\Delta^2 g) * I = I * (\Delta^2 g)$$

Requires n^2 multiplications

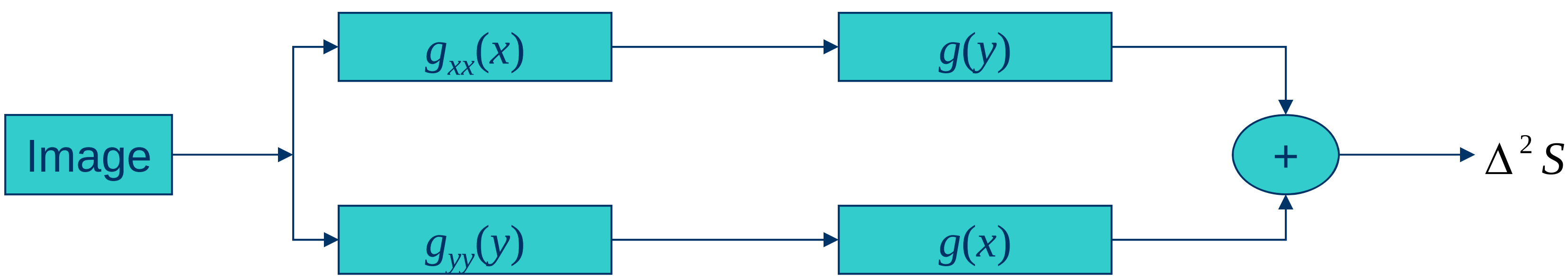
$$\Delta^2 S = (I * g_{xx}(x)) * g(y) + (I * g_{yy}(y)) * g(x)$$

Requires $4n$ multiplications

Seperability

Gaussian Filtering

Laplacian of Gaussian Filtering



Example

I

$I * \left(\Delta^2 g\right)$

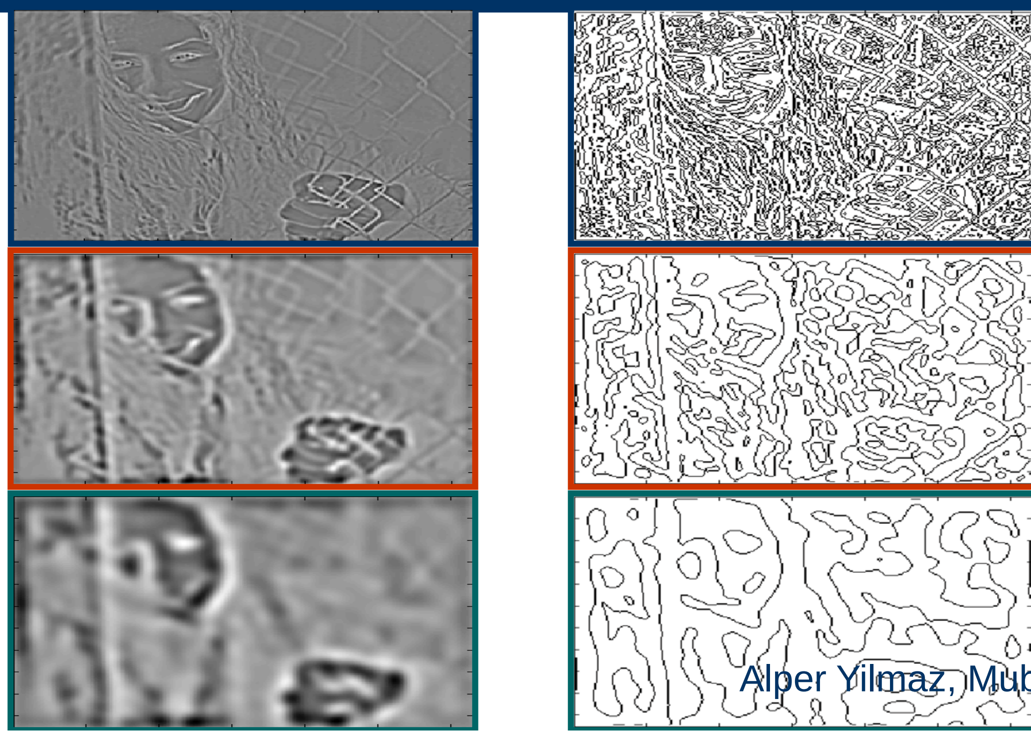
Zero crossings of $\Delta^2 S$

Example

$\sigma = 1$

$\sigma = 3$

$\sigma = 6$



Algorithm

- Compute LoG
  - Use 2D filter $\Delta^2 g(x, y)$
  - Use 4 1D filters $g(x),\ g_{xx}(x),\ g(y),\ g_{yy}(y)$
- Find zero-crossings from each row
- Find slope of zero-crossings
- Apply threshold to slope and mark edges

Quality of an Edge

- Robust to noise
- Localization
- Too many or too less responses

Quality of an Edge

Canny Edge Detector

- Criterion 1: Good Detection: The optimal detector must minimize the probability of false positives as well as false negatives.

- Criterion 2: Good Localization: The edges detected must be as close as possible to the true edges.

- Single Response Constraint: The detector must return one point only for each edge point.

Canny Edge Detector Steps

1. Smooth image with Gaussian filter
2. Compute derivative of filtered image
3. Find magnitude and orientation of gradient
4. Apply “Non-maximum Suppression”
5. Apply “Hysteresis Threshold”

Canny Edge Detector First Two Steps

- Smoothing

$$S = I * g(x,y) = g(x,y) * I$$

$$g(x,y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

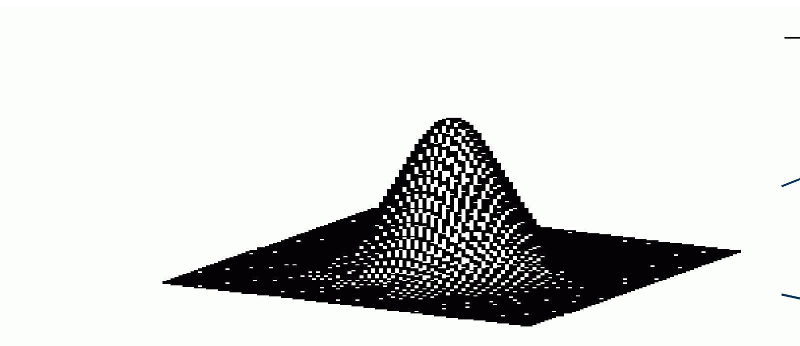
- Derivative

$$\nabla S = \nabla(g * I) = (\nabla g) * I$$

$$\nabla S = \begin{bmatrix} g_x \\ g_y \end{bmatrix} * I = \begin{bmatrix} g_x * I \\ g_y * I \end{bmatrix}$$

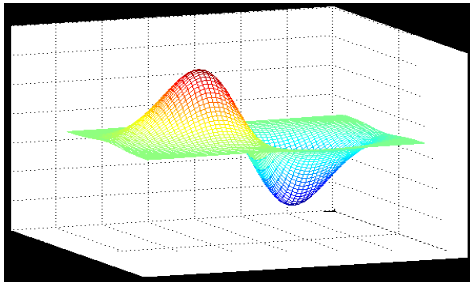
$$\nabla g = \begin{bmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} g_x \\ g_y \end{bmatrix}$$

Canny Edge Detector
Derivative of Gaussian

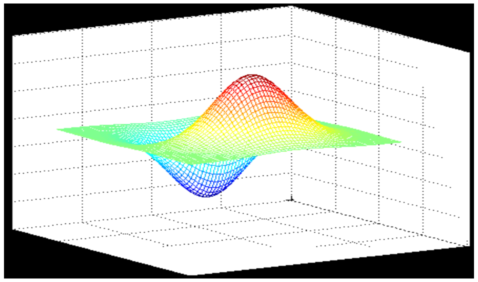


$g(x, y)$

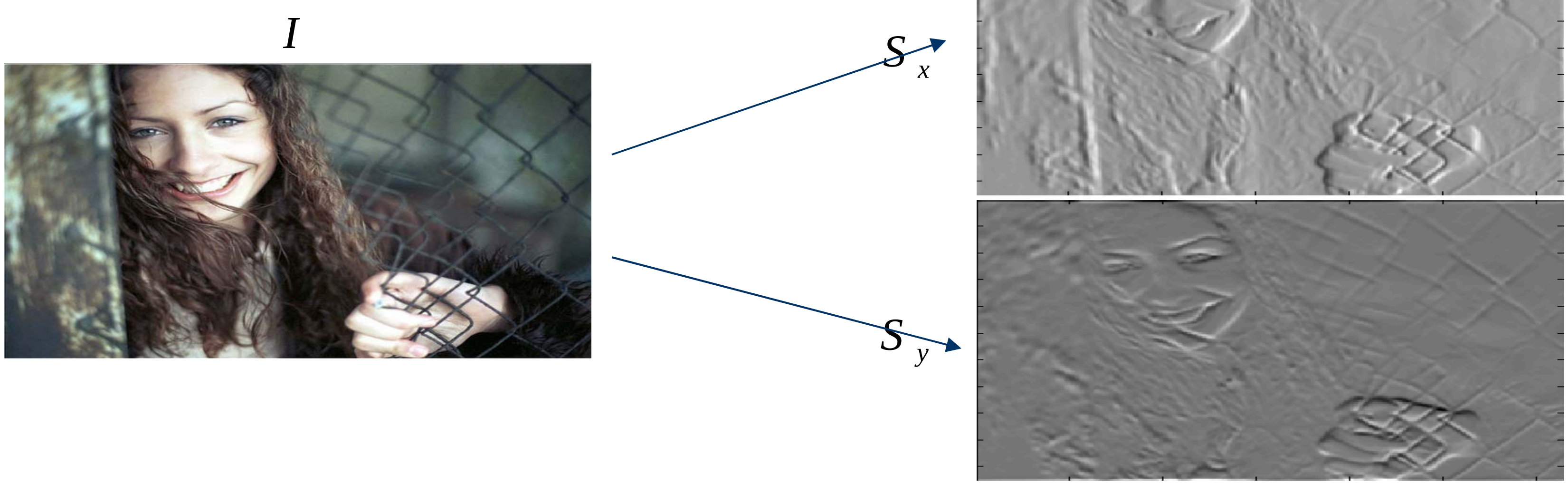
$g_x(x, y)$



$g_y(x, y)$



Canny Edge Detector
First Two Steps



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Canny Edge Detector
Third Step

- Gradient magnitude and gradient direction

(S_x, S_y) Gradient Vector

magnitude $= \sqrt{(S_x^2 + S_y^2)}$

direction $= \theta = \tan^{-1} \frac{S_y}{S_x}$

image

gradient magnitude

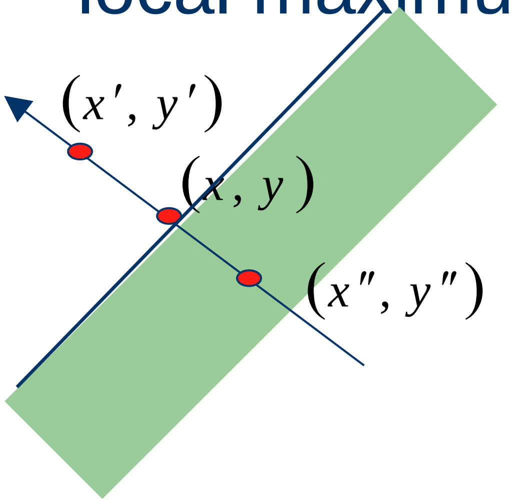
Canny Edge Detector
Fourth Step

- Non maximum suppression

We wish to mark points along the curve where the **magnitude is largest**. We can do this by looking for a maximum along a slice normal to the curve (non-maximum suppression). These points should form a curve. There are then two algorithmic issues: at which point is the maximum, and where is the next one?

Canny Edge Detector
Non-Maximum Suppression

- Suppress the pixels in $|\nabla S|$ which are not local maximum



$$M(x, y) = \begin{cases} |\nabla S|(x, y) & \text{if } |\nabla S|(x, y) > |\Delta S|(x', y') \\ & \& \ |\Delta S|(x, y) > |\Delta S|(x'', y'') \\ 0 & \text{otherwise} \end{cases}$$

x’ and **x**’’ are the neighbors of **x** along normal direction to an edge

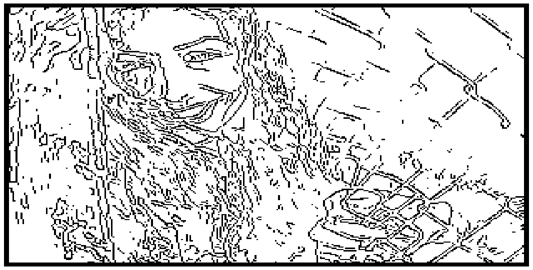
Canny Edge Detector
Non-Maximum Suppression

$$|\Delta S| = \sqrt{S_x^2 + S_y^2}$$

M

For visualization

$M \geq Threshold = 25$



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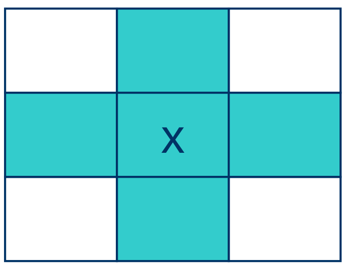
Canny Edge Detector
Hysteresis Thresholding

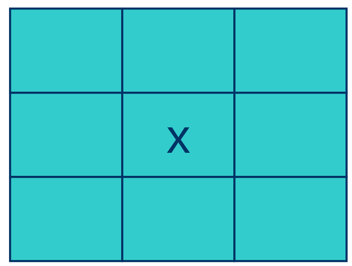
- If the gradient at a pixel is
  - above "**High**", declare it as an '**edge pixel**'
  - below "**Low**", declare it as a "**non-edge-pixel**"
  - **between** "low" and "high"
    - Consider its neighbors iteratively then declare it an "edge pixel" if it is **connected** to an 'edge pixel' **directly** or via pixels **between** "low" and "high".

Canny Edge Detector
Hysteresis Thresholding

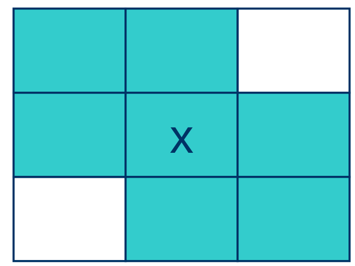
- Connectedness



4 connected

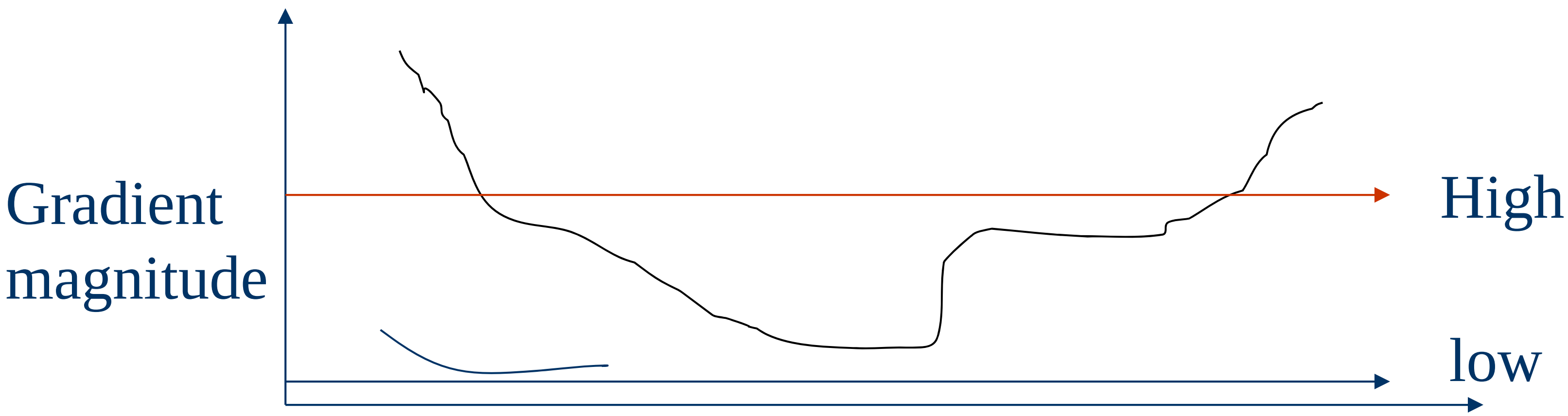


8 connected



6 connected

Canny Edge Detector
Hysteresis Thresholding



Canny Edge Detector
Hysteresis Thresholding

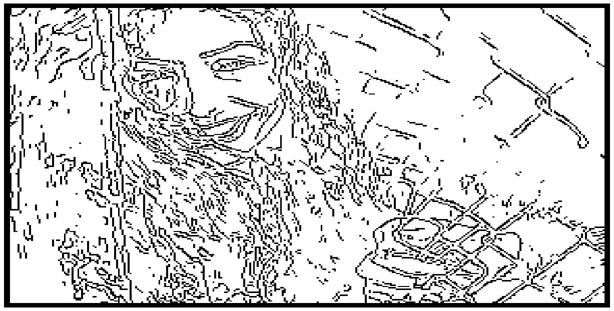
- Scan the image from left to right, top-bottom.
  - The gradient magnitude at a pixel is above a high threshold declare that as an edge point
  - Then recursively consider the *neighbors* of this pixel.
    - If the gradient magnitude is above the low threshold declare that as an edge pixel.

Canny Edge Detector
Hysteresis Thresholding

M

regular

$M \geq 25$



Hysteresis

$High = 35$

$Low = 15$

Suggested Reading

- Chapter 4, Emanuele Trucco, Alessandro Verri, "Introductory Techniques for 3-D Computer Vision"
- Chapter 2, Mubarak Shah, “Fundamentals of Computer Vision”