COT6505 Computational Methods

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- Office Hours:
  - 2PM to 3PM Mon, 4PM-5PM Tu, 3PM-4PM Thurs
- Grading
  - Mid term 25%, Final 35%, assignments (homework, programs) 40%
- Text Book
  - Numerical Optimization, Jorge Nocedal and S. Wright, Springer, 1999.
- Other suggested Books
  - Numerical Analysis, Burden and Faires, PWS Kent.
  - Numerical Recipes in C, W. Press et al, Cambridge University press.

Contents

1. Introduction
2. Fundamentals of Unconstrained optimization
3. Line Search methods
4. (chapter 5) Conjugate Gradient Methods
5. (chapter 6) Practical Newton's methods
6. (chapter 10) Non-linear least squares problems

Optimization

- People optimize
  - Stocks
  - Job
  - exam
- Convert qualitative description into quantitative function
  - Objective function
  - Variables
  - constraints

Examples

- Transportation problem
- Chess playing
- Robot path planning
- Computing the optimal shape of an automobile or aircraft
- Controlling a chemical process or a mechanical device to optimize or meet standards of robustness
- Computer Vision
  - Camera Pose estimation
  - Optical flow
  - Stereo depth estimation
  - etc

Optimization

- Minima, maxima or zero of a function
- Local minima vs global minima

Optimization Problems

- Single variable
- Multiple variables
- Linear
- Non-linear
- Unconstraint optimization
- Constraint optimization

$$\min (x_1 - 2)^2 + (x_2 - 1)^2$$

$$\text{subject to} \left\{ x_1^2 - x_2 \le 0, x_1 + x_2 \le 2 \right.$$

Desirable Properties

- Robustness
- Accuracy
- Efficiency

Solution

- Iterative solution $X^0, X^1, X^2, \ldots X^n$
  - Initial estimate $X^n \approx X^{n-1}$
  - Convergence $X^n \approx P$
    - Linear
    - Super linear
    - Quadratic

Numerical Optimization

- Computation of
  - derivatives,
  - gradient,
  - Jacobian,
  - Hessian
- Analytical derivatives not possible
- Numerical derivatives, finite difference
- Solution of a linear system (Inverse of a matrix)

$$\text{Derivative}: \; f'(x) = \frac{df}{dx}, \; x \text{ is a scalar}$$

$$\text{Gradient}: \; \nabla f(x_1, x_2, \ldots, x_n) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}\right)$$

Jacobian

$$F(x_1, x_2, \ldots, x_n) = f_1(x_1, x_2, \ldots, x_n), f_2(x_1, x_2, \ldots, x_n), \ldots, f_m(x_1, x_2, \ldots, x_n)$$

$$J(F) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \cdots & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

Hessian

$$f(x_1, x_2, \ldots, x_n)$$

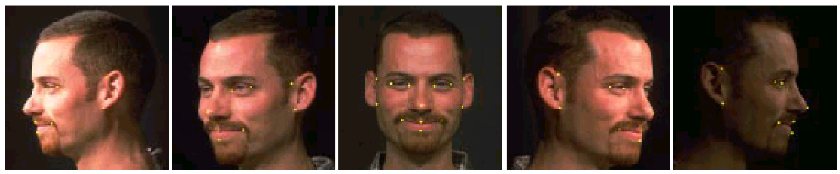
$$H(f) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1 \partial x_1} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \cdots & \cdots & \frac{\partial^2 f}{\partial x_n \partial x_n} \end{bmatrix}$$

Optimization Methods

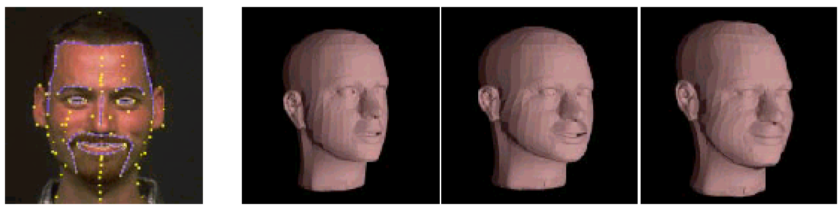
- Gradient Descent
- Conjugate Gradient
- Newton
- Quasi Newton
- Levenberg Marquadet

Real world Examples

Synthesizing Realistic Facial Expressions



(a)



Synthesizing Realistic Facial Expressions

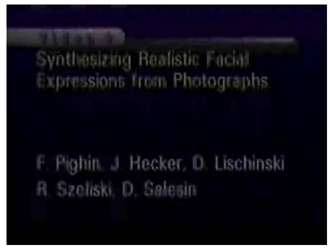
$$x_i'^k = s_k \frac{\mathbf{r}_x^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_X^k}{1 + \eta^k \mathbf{r}_z^{\mathbf{k}} \mathbf{p}_{\mathbf{i}}}$$

$$y_i'^k = s_k \frac{\mathbf{r}_y^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_Y^k}{1 + \eta^k \mathbf{r}_z^{\mathbf{k}} \mathbf{p}_{\mathbf{i}}} \quad w_i^k = (1 + \eta^k (\mathbf{r}_z^k . \mathbf{p}_i))^{-1}$$

$$w_i^k (x_i'^k + x_i'^k \eta^k (\mathbf{r}_{\mathbf{z}}^{\mathbf{k}} . \mathbf{p}_{\mathbf{i}}) - s^k (\mathbf{r}_X^{\mathbf{k}} . \mathbf{p}_{\mathbf{i}} + T_X^k)) = 0$$

$$w_i^k (y_i'^k + y_i'^k \eta^k (\mathbf{r}_{\mathbf{z}}^{\mathbf{k}} . \mathbf{p}_{\mathbf{i}}) - s^k (\mathbf{r}_Y^{\mathbf{k}} . \mathbf{p}_{\mathbf{i}} + T_Y^k)) = 0$$

Synthesizing Realistic Facial Expressions



Computing Projective Transformation

$$x' = \frac{a_1 x + a_2 y + b_1}{c_1 x + c_2 y + 1}$$

Projective

$$y' = \frac{a_3 x + a_4 y + b_2}{c_1 x + c_2 y + 1}$$

$$E = \sum [f(x', y') - f(x, y)]^2 = \sum e^2$$

min

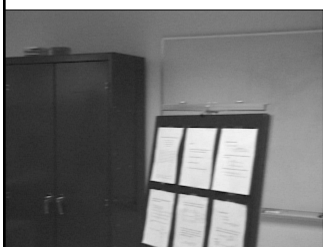
Computing Projective Transformation

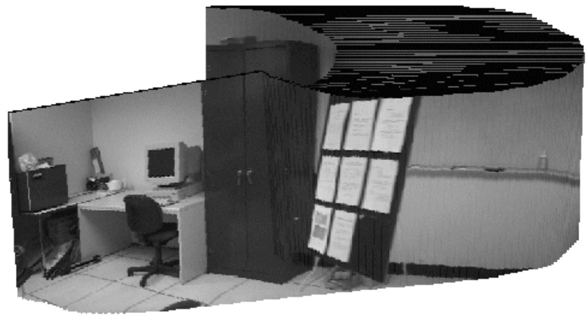
Motion Vector:

$$\mathbf{m} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & b_1 & b_2 & c_1 & c_2 \end{bmatrix}^T$$

Video Mosaic

Video Mosaic





Video Mosaic