Detecting Time-Varying Corners

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The algorithms for *structure from motion* require solution of the correspondence problem. By detecting only time-varying tokens, the problem may be significantly simplified. In this paper, a time-varying corner detector is described which is based on the **and** operation between the cornerness and the temporal derivative. It is shown that the corner detectors by Zuniga and Haralick (*IEEE CVPR Conf. 1983*, pp. 30–37), Kitchen and Rosenfeld (*Pattern Recognition Lett.* 1, 1982, 95–102), and Dreschler and Nagel (*Proc. IJCAI, 1981*, pp. 692–697) are equivalent. In this time-varying corner detector, the Zuniga and Haralick, in loc. cit., corner detector is used for finding the cornerness at a point and the absolute value of difference in intensity at a point is used to approximate the temporal derivative. The results of the time-varying corner detector for the the real scenes and the synthetic images with random background and random object are shown. © 1984 Academic Press, Inc.

1. INTRODUCTION

Time-varying features play an important role in dynamic scene analysis. Biological systems are capable of detecting features in the environment which are changing with time. The features may be varying either due to the motion of an object or the observer. The human vision system utilizes both of these means to perceive the world. Certain objects become clear when we move our eyes, head, or body [3]. Similarly the perception is improved when the objects move.

It is well known that the structure of an object can be determined from its motion. In various approaches to *structure from motion* [12, 9, 11], correspondence of N points in M frames is required. The correspondence problem is solved by detecting *interesting points* in frames. Corners are considered good candidates for establishing correspondence [2]. In the proposed approaches for *structure from motion* the corners are detected in each frame. Since each frame may contain many stationary corners, in addition to a few moving corners, the correspondence becomes more difficult. If only moving corners could be detected, then the matching of corners from frame to frame may become more tractable and computationally efficient. Our efforts to use corners in each frame to determine motion parameters [6] showed the difficulty in working with corner detectors and encouraged us to investigate the possibility of detecting only time-varying corners.

Another application of time-varying corners may be in optical flow computation in conjunction with Nagel's approach. Nagel showed that displacement vectors may be computed, using only local intensity values, at grey level corners [8]. This is not surprising because at the corners the aperture problem does not exist. As shown in Fig. 1, it is possible to use local methods to compute displacement components at a corner. Nagel also developed a method to propagate displacement vectors at corners

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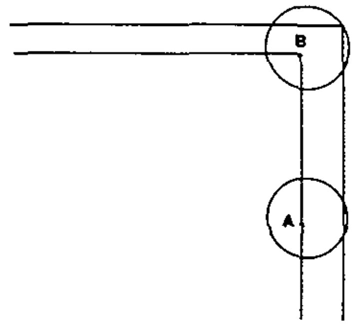

FIG. 1. Aperture Problem. Local information at point A is not sufficient to give the displacement information. At B the local information is sufficient to give the displacement of the object.

to other points in the image. The computation of displacement vectors may be more efficient and correct if only time-varying corners are used in the computation.

Haynes and Jain [4] proposed a time-varying edge detector, which is simple, fast, and noise insensitive. The time-varying edge detector should find the points which are edges and which are moving. Therefore, they proposed that the **and** operator be used to combine edgeness and motion, which is equivalent to the multiplication. The operator is given as

$$E_t(x, y, t) = \frac{\partial f(x, y, t)}{\partial t} * G(f(x, y, t)).$$

Where $f(x, y, t)$ is a frame at time t, E_t is the time varying edgeness, G is any edge operator (e.g., 3×3 Sobel) and $\partial f / \partial t$ is temporal derivative.

An important property of this operator is that it can detect the edges which are weak but have significant motion and the edges which are strong but have weak motion. The efficacy of this edge operator was demonstrated using several scenes.

In this paper we present a time-varying corner detector which gives good results even in images having the random background and random object environment. In the next section we will discuss three gray level corner detectors proposed in the recent years and we will show that all three are essentially equivalent. In Section 3 the time-varying corner detector is described. In Section 4 we present the results for the real and synthetic scenes with the random background and random object.

2. CORNER DETECTION

Many corner detectors have been reported in the literature [10]. In the earlier approaches, first the image is segmented and then the curvature of the boundary of the object is computed. If that curvature is above some threshold then that point is declared as the corner point. Since the segmentation is far from being perfect, these approaches do not seem to work properly with real scenes. Recently, gray level based corner detectors have been reported which give good results. The attractive point in the gray level based corner detectors is that they do not rely on the prior segmentation. Instead, the segmentation can be made easy by using corners detected [7].

The effort to detect the corners based on the gray levels was made by Beaudet [1]. Beaudet used the DET operator to detect the saddle points at which the gray level function $g(x, y)$ is neither maximum nor minimum. Beaudet considers the second-order Taylor series of function $g(x, y)$ to compute

$$DET = g_{xx}g_{yy} - g_{xy}^2, \tag{1}$$

where g_{xx}, g_{yy}, and g_{xy} are the second-order partial derivatives. This operator finds the corners on the both sides of the edge.

The three popular gray level corner detectors are: Zuniga and Haralick [13], Kitchen and Rosenfeld [7], and Dreschler and Nagel [2]. It can be shown that all three corner detectors are equivalent. In fact Nagel [8] has shown that the Kitchen and Rosenfeld corner detector is similar to the Dreschler and Nagel corner detector. We will show here that, in principle, the Kitchen–Rosenfeld corner detector is equivalent to the Zuniga–Haralick corner detector.

2.1. Zuniga–Haralick Corner Detector

The Zuniga–Haralick corner detector is based on Haralick and Watson's gray level facet model [5]. In this model they fit the bicubic polynomial to the gray level function. The polynomial used by them is

$$\begin{aligned} g(x, y) = {} & k_1 + k_2x + k_3y + k_4x^2 + k_5xy + k_6y^2 \\ & + k_7x^3 + k_8x^2y + k_9xy^2 + k_{10}y^3. \end{aligned} \tag{2}$$

Haralick and Watson consider the $n * n$ neighborhood around each pixel and use a least-square fit to find the coefficients k_i's in (2). For each pixel they find out the first and second directional derivatives of function $g(x, y)$. The derivatives are taken in the direction (g_x, g_y) which is equal to the gradient direction. The pixel is declared an edge point if the first derivative is above some threshold and the second derivative is zero.

The corner detector works as follows: at each pixel location the rate of change of the gradient angle of the fitted function $g(x, y)$ is computed, which gives a measure of the cornerness. If at any pixel location this cornerness value is above a preset threshold and that point is an edge point then that pixels is declared as a corner. The gradient angle θ is given by

$$\tan\theta = \frac{g_x}{g_y}. \tag{3}$$

Zuniga and Haralick find the directional derivative of (3) in the direction α equal to $(-g_y, g_x)$ which is orthogonal to the gradient direction (g_x, g_y). Evaluating the derivative of θ at origin and using Eq. (2) for the function $g(x, y)$ they get the expression

$$K = \frac{-2\left(k_2^2k_6 - k_2k_3k_5 + k_3^2k_4\right)}{\left(k_2^2 + k_3^2\right)^{1.5}}, \tag{4}$$

where K is the rate of change of θ which gives the measure of cornerness at the point (x, y).

2.2. Kitchen–Rosenfeld Corner Detector

Kitchen and Rosenfeld [7] project the change of gradient direction vector (θ_x, θ_y) along an edge and multiply the result by the local gradient magnitude. Kitchen and Rosenfeld give the following expression for cornerness:

$$K = \frac{g_{xx}g_y^2 + g_{yy}g_x^2 - 2g_{xy}g_x g_y}{g_x^2 + g_y^2}. \tag{5}$$

Kitchen and Rosenfeld consider the quadratic polynomial for the gray level function $g(x, y)$, but, for the sake of the comparison, let us consider the bicubic polynomial as considered by Haralick and Watson [5]. If we now evaluate all the terms in (5) considering the polynomial given in Eq. (2) at (0, 0) and substitute back in (5), we get the following expression for the cornerness:

$$K = \frac{-2\left(k_2^2 k_6 - k_2 k_3 k_5 + k_3^2 k_4\right)}{\left(k_2^2 + k_3^2\right)}. \tag{6}$$

2.3. Dreschler–Nagel Corner Detector

Dreschler and Nagel also consider the function $g(x, y)$ and approximate it with the second-order Taylor expansion. They use Beaudet's operator as given in Eq. (1). Since Beaudet's operator gives corner on both sides of the edge, Dreschler and Nagel apply the following algorithm to eliminate the false corners:

(i) Determine $g_{xx}g_{yy}$ called Gaussian curvature.

(ii) Select locations of extremal—positive as well as negative—Gaussian curvature.

(iii) Match a location of maximum positive Gaussian curvature such as P with a location of extreme negative Gaussian curvature such as B provided that the directions of those principal curvatures, i.e., g_{xx} and g_{yy} which have opposite sign at B and P are approximately aligned.

(iv) Select point T where the principal curvature crosses zero. This corresponds to the corner point.

2.4. Equivalence of Three Corner Detectors

Nagel [8] has shown that their corner detector is equivalent to the Kitchen–Rosenfeld corner detector. We will show in this section that the Zuniga–Haralick and Kitchen–Rosenfeld methods are essentially equivalent. The only difference between the two expressions for cornerness K, given by Zuniga and Haralick and Kitchen and Rosenfeld, in [4, 5], respectively, is the factor $(g_x^2 + g_y^2)^{0.5}$ which is the gradient magnitude. The gradient magnitude can be considered as the measure of the edgeness. By multiplying the rate of change of gradient direction with the gradient magnitude, Kitchen and Rosenfeld introduce the and operation between cornerness and edgeness. In the Zuniga–Haralick corner detector, the edgeness condition is explicit, i.e., the point is declared as a corner if:

(1) cornerness is above some threshold and

(2) it is an edge point.

In other words, Zuniga and Haralick first detect the edges, then they compute the cornerness at each point and apply the above two conditions to detect the corners. An important point to note here is that due to condition (2), the Zuniga–Haralick corner detector eliminates all false corners in the background which might appear due to noise. The other two corner detectors might not be able to eliminate those false corners.

When the edge near the corner is blurred, the Kitchen–Rosenfeld corner detector does not give good results due to the inefficiency of gradient magnitude as a measure of edgeness. In such a situation, the corner detector responds all the way across the edge. They solve this problem by using the heuristics of nonmaximum suppression to edge magnitudes. Since Haralick and Watson's zero crossing edge detector based on their facet model gives significantly better results than the gradient edge operator, the Zuniga–Haralick corner detector does not have problems with the blurring.

Thus we can conclude that as far as the cornerness measure is concerned, the Kitchen–Rosenfeld and the Zuniga–Haralick corner detectors are the same in principle; they differ only with respect to the edgeness measure and steps in implementation. Zuniga and Haralick get significantly better results due to the high quality of edges obtained by using the facet model and the fact that first edges are determined and then the cornerness is computed only at edge points.

3. TIME VARYING CORNERS

The time-varying corner detector is based on the **and** operation between the cornerness and the temporal derivative

$$C_t = C_s * \nabla,$$

where C_t is the time varying cornerness, C_s is static cornerness, and ∇ is a measure of the temporal variations at the point.

Consider two frames f1 and f2 of a sequence shown in Fig. 2. There is one object which is displaced in the frame f2 with respect to f1. The aim is to detect the corners of the object that have been displaced. We apply the Zuniga and Haralick corner detector to f1 which gives C_s, the cornerness at each pixel. The time-varying operator ∇ finds the regions in f1 which have changed in the gray level characteristics. For the time-varying operator ∇ there are two approaches: temporal derivative and likelihood ratio, which we will discuss below. After applying some threshold to the product C_t the time varying corners can be detected.

3.1. *Likelihood Ratio*

The likelihood ratio gives an idea about the change in the gray level distribution around some given neighborhood of a pixel location. This can be used to detect the changes, due to moving objects in the gray levels between two frames. The likelihood ratio λ is defined as

$$\lambda = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\left(\frac{\mu_1 - \mu_2}{2}\right)^2\right)^2 \Big/ (\sigma_1 * \sigma_2), \tag{7}$$

where $\sigma_1, \mu_1, \sigma_2, \mu_2$ are the variance and mean in frame 1, and the variance and mean of the corresponding pixels in frame 2, respectively.

a

255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	100c	100a	100	100	255	255	255	255	255	255
255	255	100	100	100	100	255	255	255	255	255	255
255	255	100	100	100	100	255	255	255	255	255	255
255	255	100	100	100	100	255	255	255	255	255	255
255	255	100	100	100	100	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
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255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255

b

255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255c	255a	255	255	100	100	100	100	255	255
255	255	255	255	255	255	100	100	100	100	255	255
255	255	255	255	255	255	100	100	100	100	255	255
255	255	255	255	255	255	100	100	100	100	255	255
255	255	255	255	255	255	100	100	100	100	255	255
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255	255	255	255	255	255	255	255	255	255	255	255
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255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255

FIG. 2. (a) Frame f1; (b) Frame f2. The 3 * 3 neighborhood around the actual corner **c** in frame f1 contains 4 pixels having value 100 while around point **a** there are 6 pixels having value equal to 100. Therefore the change, in graylevel distribution, at point **a** in frame f2, with respect to frame f1, is more than the change at point **c**.

Consider Fig. 2 where we have shown two frames f1 and f2. The object is represented by the gray level 100 while background by 255. It is shown that the object has moved in frame f2 with respect to f1. Now let us find λ at corner point **c** and the point **a** next to it by considering a 3×3 neighborhood as shown in Figs. 2a and b. In the considered neighborhood of point **c** in frame f1, there are four pixels which have gray levels equal to 100 while the remaining five pixels have gray levels equal to 255. In frame f2 all the pixels in the neighborhood corresponding to the location of **c** have values equal to 255. Now consider the 3×3 neighborhood around the point **a** close to point **c** in frames 1 and 2. It is easy to see that six pixels in f1 have the value 100 while the remaining three have the value 255. In f2 each pixel in the corresponding neighborhood is equal to 255.

It is clear from the above that there is more change in the gray level distribution at point **a** than at the corner point **c**. Therefore the λ for **a** will be greater than **c**. Since C_s for points **c** and **a** do not differ much, after multiplying with λ it will be hard to distinguish between points **c** and **a**.

Therefore if a likelihood ratio is used then it will be hard to eliminate the false corner point. This implies that the likelihood ratio is not suitable for time-varying corner detection; it will give blurred corners.

3.2. *Temporal Derivative*

The derivative in the discrete domain can be approximated by the difference operation. Consider two frames f1 and f2 that we considered in the previous section. The temporal derivative is approximated by taking the difference between the gray level of a pixel in f1 and the corresponding gray level of a pixel in f2. The picture thus obtained is called the difference picture. It is easy to see that the entries in the difference picture will be significant only at the pixel location where the object has moved. Moreover, this difference is the same for each object pixel. It should be mentioned here that better results may be obtained by computing the temporal derivative from more than two frames. In this paper, we report our experiments with only two frames of a sequence, however.

Note the difference between the temporal derivative and the likelihood ratio. In the difference picture, each pixel contributes to itself only. While in the likelihood ratio, many neighborhood pixels also contribute. Due to this fact, there is always competition between the corner points and the points close to them in the likelihood ratio approach. But this problem is not encountered in the temporal derivative approach.

4. RESULTS

In order to study the efficacy of the proposed corner detector, we applied it to several synthetic and laboratory generated scenes. The synthetic scenes had object and background with random texture and significant amount of noise added. We report here results for one of the more difficult scenes. Our experiments with less noisy scenes showed very good determination of moving corner points.

The results of time varying corner detector are shown in Figs. 3–5. In Fig. 3 the pictures considered are made of the random background and the random object. We added 30% uniform random noise to both the background and the object. The background gray level is a random number between 0 to 255 and the object gray level is between 0 to 80. Due to the added noise and nonuniform object and the background, the time-varying corners detected by our operator are expanded to ± 1 pixels around their actual locations. As it is clear from Fig. 3, even in case of such extreme noise, good time-varying corners were detected by this approach.

In another experiment with a synthetic scene, both the background and the moving square were drawn from the same range of random numbers. The random background and the random object reminds us of the famous Bradick's illusion. In

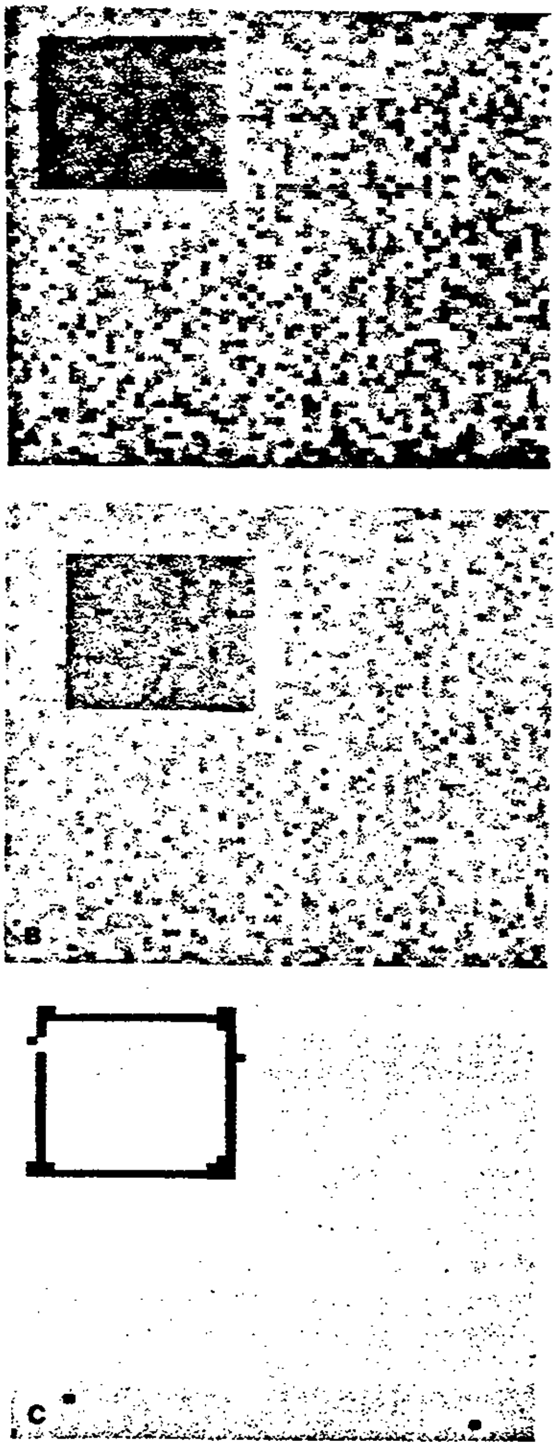

FIG. 3. (A): Frame f1, background is a random number between 0 to 255, and object is between 0 to 80. Uniform noise equal to 30% is added to the background and the object. (B): Frame f2, the object moves 3*3 pixels. (C): The time varying corners detected by the operator superimposed in the edge image.

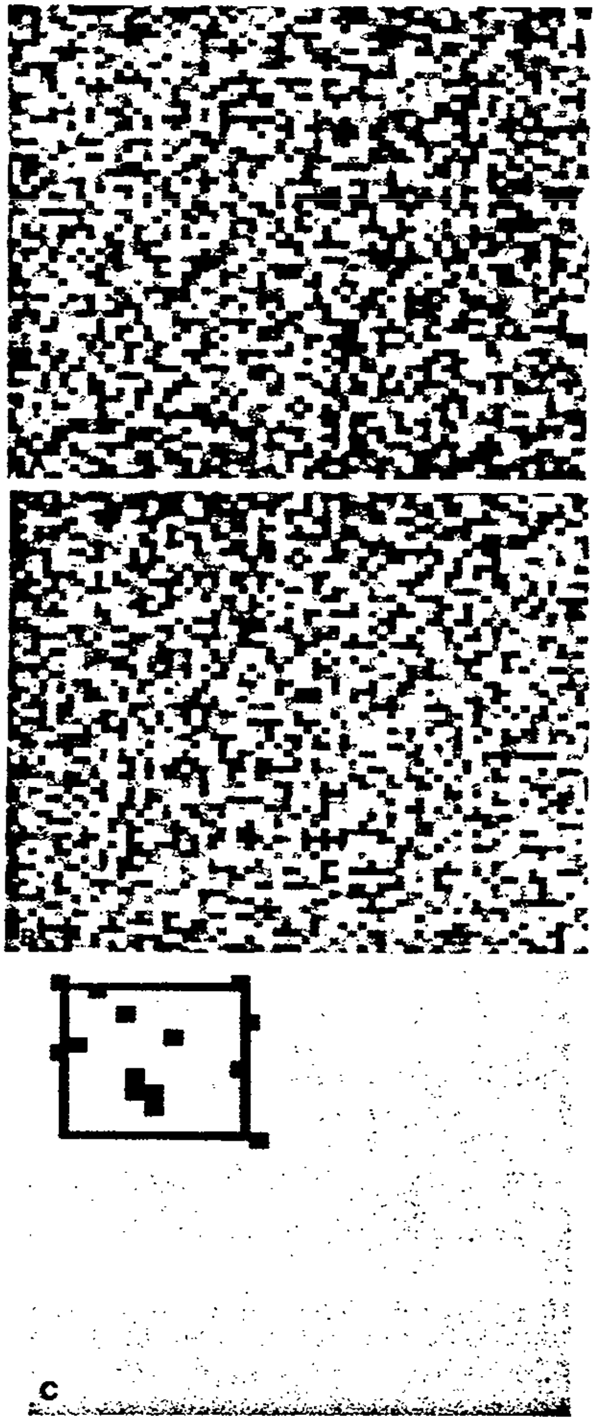

FIG. 4. (A): Frame f1, background and the object is a random number between 0 to 255. (B): The object moves 3 • 3 pixels. (C): The time varying corners detected by the operator, there are some false corners in side the object. But the operator is able to distinguish between the background and the object.

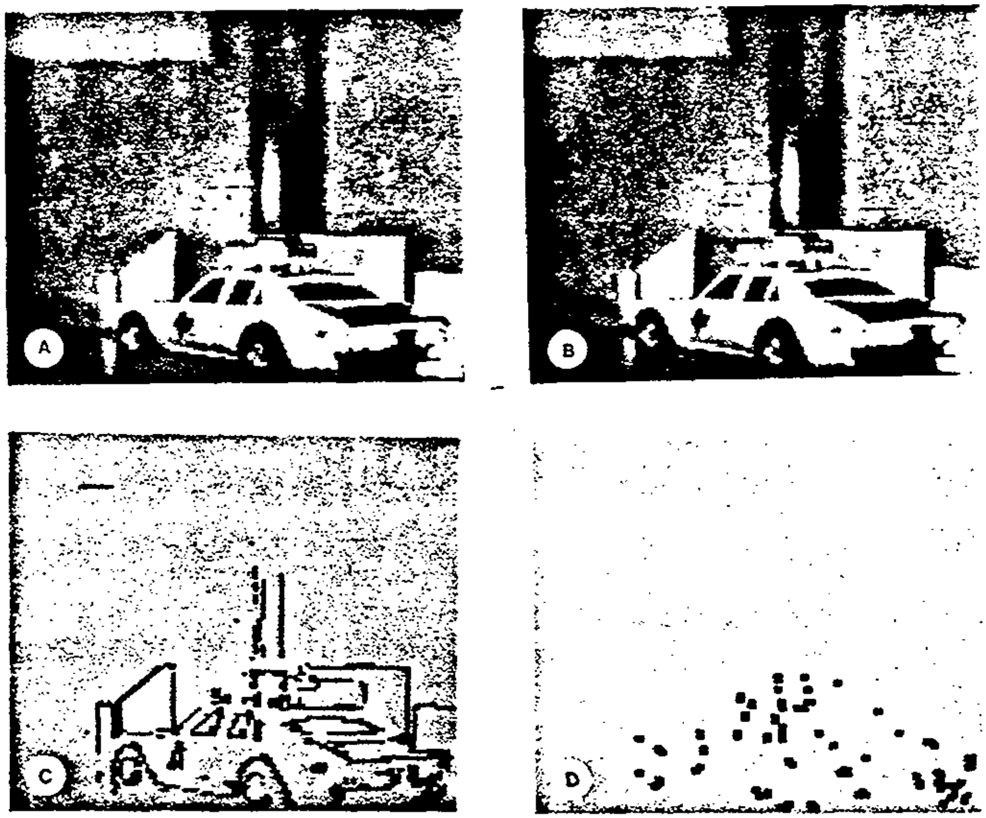

FIG. 5. (A): Frame f1, a toy car moving towards left. (B): Frame f2, the car moves with respect to frame f1. (C): The time-varying corners detected by the operator. (D): The time-varying corners superimposed on the edges.

our experiments we have verified this illusion by simulating the random object and the background in two frames with object displaced in the second frame (see Fig. 4). From one display it it impossible to tell where the object is. But the location of the object becomes obvious when the two frames are displayed successively. Our corner detector gives time-varying corners in this case as shown in Fig. 4. Note that several corners are detected within the square also. These are the points of the background that are gray value corners *and* their intensity has changed due to the displacement of the random object. These points are correct time-varying, though not moving, corner points.

In Fig. 5 we have shown two frames of a moving toy car. The time-varying corners detected by our corner detector are shown in Fig. 5c. The time-varying corners superimposed on edges are given in Fig. 5d. Some corners adjacent to the right rear wheel are not detected due to the small area of the neighborhood we considered to fit the fact model.

5. CONCLUSION

In this paper, we proposed a time-varying corner detector which gives good results even in the random background and the random object environment with significant noise. We have shown that though the corner detectors by Zuniga and Haralick, Dreschler and Nagel, and Kitchen and Rosenfeld are equivalent in principle, the Zuniga and Haralick corner detector gives better results due to a powerful edgeness measure.

We believe that the correspondence problem in *structure from motion* will be significantly simplified using this corner detector. It appears that the time-varying corners may also be useful in Nagel's approach for the computation of optical flow. Though we have not yet integrated our corner detector in a system for recovering *structure form motion*, we intend to do experiments to see the effectiveness of time-varying corners in real world dynamic scene analysis.

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